

MBMT Number Theory Round – Leibniz

March 30, 2019

Full Name _____

Team Number _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- _____ 1 On Jupiter, a day is 10 hours long. Jim is a strange animal on Jupiter who eats a rock every 3 hours. Exactly at midnight, Jim eats a rock. How many hours will pass before he eats again at midnight?
- _____ 2 What is the remainder when $2017^2 + 2018^2 + 2019^2 + 2020^2 + 2021^2$ is divided by 5?
- _____ 3 We call a year *summable* if there exists some day during the year such that the sum of the month and the day equals the last two digits of the year. Find the first year after 2018 that is not summable.
- _____ 4 Find the largest multiple of 4 that has fewer than six positive integer factors.
- _____ 5 Define a real number x to be *imbalanced* if its decimal expansion is infinite and, in the decimal expansion of x , all but a finite number of digits have the same nonzero value. For instance, 0.123 is not imbalanced since its decimal expansion is finite. What is the smallest n such that, for all real numbers x , at least one of $x, 2x, \dots, nx$ is **not** imbalanced?
- _____ 6 The number $\frac{1}{2310}$ can be written in the form $\frac{1}{a} - \frac{1}{b}$, where a and b are positive integers and $a + b$ is as small as possible. Find $b - a$.
- _____ 7 Find the number of positive integers $k \leq 100$ such that $6^k + 2 \cdot 5^k - 1$ is divisible by 31.
- _____ 8 Let a *pegboard* be a (possibly rotated) infinite square grid of points with side length 1 in the plane. Call two pegboards *equivalent* if one is a translation of another. Let the *gap* between two pegboards be the minimum distance between a point in each pegboard. Find the maximum gap between two non-equivalent pegboards.