MBMT Leibniz Guts Round — Set 1

March 30, 2019

1 **[3]** Find the units digit of $3^{1^{3^{3^7}}}$.

Proposed by Kevin A. Zhou

Solution. $\boxed{3}$

Since $1^k = 1$ for positive k, the expression is equal to $3^1 = \boxed{3}$. $\square$

2 **[3]** A standard deck of cards contains 13 cards of each suit (clubs, diamonds, hearts, and spades). After drawing 51 cards from a randomly ordered deck, what is the probability that you have drawn an odd number of clubs?

Proposed by Jacob Stavrianos

Solution. $\boxed{\dfrac{3}{4}}$

The problem is asking for the probability that the last card remaining is not a club. Since one-fourth of the cards are clubs, that is $\boxed{\frac{3}{4}}$. $\square$

3 **[3]** Square $ABCD$ with side length 1 is rolled into a cylinder by attaching side AD to side BC. What is the volume of that cylinder?

Proposed by Daniel Zhu

Solution. $\boxed{\dfrac{1}{4\pi}}$

The height is 1. The circumference is 1, so the radius is $\frac{1}{2\pi}$. Thus, the volume is $\boxed{\frac{1}{4\pi}}$. $\square$

4 **[3]** Haydn is selling pies to Grace. He has 4 pumpkin pies, 3 apple pies, and 1 blueberry pie. If Grace wants 3 pies, how many different pie orders can she have?

Proposed by Steven Qu

Solution. $\boxed{7}$

There are 4 non-blueberry pie orders and 3 blueberry pie orders. $\square$

5 **[3]** Kevin has written 5 MBMT questions. The shortest question is 5 words long, and every other question has exactly twice as many words as a different question. Given that no two questions have the same number of words, how many words long is the longest question?

Proposed by Jacob Stavrianos

Solution. $\boxed{80}$

The questions have length 5, 10, 20, 40, and $\boxed{80}$. □

MBMT Leibniz Guts Round — Set 2

March 30, 2019

____________ **6** **[4]** Alex has 100 Bluffy Funnies in some order, which he wants to sort in order of height. They're already *almost* in order: each Bluffy Funny is at most 1 spot off from where it should be. Alex can only swap pairs of adjacent Bluffy Funnies. What is the maximum possible number of swaps necessary for Alex to sort them?

Proposed by Jacob Stavrianos

Solution. $\boxed{50}$

Let $f(n)$ be the the minimum possible number of swaps for a given almost-ordered sequence, except with n instead of 100.

Now, define Bluffy Funny i as the Funny that is i^{th} in the correct ordering. We notice that Funny 1 is in spot 1 or 2. If it's in spot 1, then $f(n) = f(n-1)$, since the 1 spot is already correct. If it's in spot 2, then Funny 2 must be in spot 1. So we swap spots 1 and 2, getting $f(n) = f(n-2) + 1$.

We're maximizing $f(n)$, so we only consider the $f(n) = f(n-2)+1$ case. $f(0) = 0$, so we get $f(2) = 1$, $f(4) = 2$, and so on until $f(2n) = n$. From this, we get $f(100) = 50$. □

____________ **7** **[4]** Let $s(n)$ be the sum of the digits of n. Let $g(n)$ be the number of times s must be applied to n until it has only 1 digit. Find the smallest n greater than 2019 such that $g(n) \neq g(n+1)$.

Proposed by Steven Qu

Solution. $\boxed{2025}$

Note that for all $2020 \leq k \leq 2025$, $s(k) < 10$, so $g(k) = 1$. However, $s(2026) = 10$, so $g(2026) = 2$. □

____________ **8** **[4]** In the Montgomery Blair Meterology Tournament, individuals are ranked (without ties) in ten categories. Their overall score is their average rank, and the person with the lowest overall score wins.

Alice, one of the 2019 contestants, is secretly told that her score is S. Based on this information, she deduces that she has won first place, without tying with anyone. What is the maximum possible value of S?

Proposed by Daniel Zhu

Solution. $\boxed{1.4}$

Alice's rank is the sum of her ranks divided by 10. Therefore, note that her rank is $\frac{k}{10}$, where k is an integer. If Alice has $S = 1.5$, then another contestant may also have $S_1 = 1.5$. For example, both may have 5 1st places and 5 2nd places. This is the "equality" case, so $S = 1.4$ must guarantee Alice's victory. □

9 **[4]** Let A and B be opposite vertices on a cube with side length 1, and let X be a point on that cube. Given that the distance along the surface of the cube from A to X is 1, find the maximum possible distance along the surface of the cube from B to X.

Proposed by Daniel Zhu

Solution. $\boxed{\sqrt{2}}$

We want X to "point away" from B, which is best done by having X be a vertex adjacent to A. Then it is easy to see that the distance from X to B is $\sqrt{2}$. □

10 **[4]** Given the following system of equations where x, y, z are nonzero, find $x^2 + y^2 + z^2$.

$$x + 2y = xy$$
$$3y + z = yz$$
$$3x + 2z = xz$$

Proposed by Kevin A. Zhou

Solution. $\boxed{56}$

First, multiply each equation by the appropriate variable so that the right hand side becomes xyz:

$$xz + 2yz = xyz$$
$$3xy + xz = xyz$$
$$3xy + 2yz = xyz$$

Setting the first two equations equal to each other, we get $xz + 2yz = 3xy + xz$, which simplifies to $2z = 3x$. Similarly, the last two equations yield $x = 2y$. Plugging this back into $x + 2y = xy$, we get $x = 4, y = 2, z = 6$. Thus, the answer is $\boxed{56}$. □

MBMT Leibniz Guts Round — Set 3

March 30, 2019

____________ **11** **[5]** How many triples of nonnegative integers (x, y, z) satisfy the equation $6x + 10y + 15z = 300$?

Proposed by Jacob Stavrianos

Solution. $\boxed{66}$

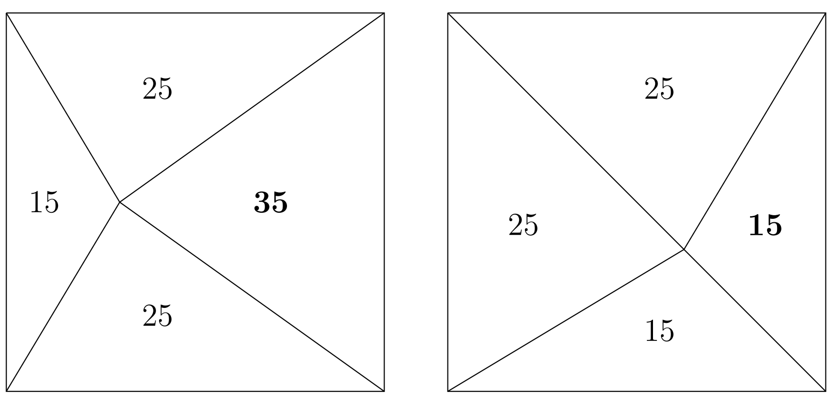
We can verify that $2 \mid z, 3 \mid y, 5 \mid x$, from which we can prove that $30 \mid 6x, 10y, 15z$. Thus, the problem is equivalent to the number of ways to distribute 10 indistinguishable "blocks" of 30 between $6x, 10y, 15x$, which by stars and bars is $\binom{12}{2} = \boxed{66}$. □

____________ **12** **[5]** Point P inside square $ABCD$ is connected to each corner of the square, splitting the square into four triangles. If three of these triangles have area 25, 25, and 15, what are all the possible values for the area of the fourth triangle?

Proposed by Steven Qu

Solution. $\boxed{15, 35}$

Let the side length of the square be s. Then it is straightforward to show that the sum of the areas of two opposite triangles are $s^2/2$. Therefore, the sum of two of the areas must equal the sum of the other two. Casework leads to two solutions:



□

____________ **13** **[5]** Tse and Cho are playing a game. Cho chooses a number $x \in [0, 1]$ uniformly at random, and Tse guesses the value of $x(1 - x)$. Tse wins if his guess is at most $\frac{1}{50}$ away from the correct value. Given that Tse plays optimally, what is the probability that Tse wins?

Proposed by Jacob Stavrianos

Solution. $\boxed{\frac{2}{5}}$

Tse should guess around where the function $x(1-x)$ is not changing much. The graph suggests that Tse should guess 0.23, so that he wins if $0.21 \le x(1-x) \le 0.25$. With some algebra (or by inspection), this occurs when $0.3 \le x \le 0.7$. Thus, the probability of Tse winning is $\boxed{\frac{2}{5}}$. □

______ **14** **[5]** Hungryman starts at the tile labeled "S". On each move, he moves 1 unit horizontally or vertically and eats the tile he arrives at. He cannot move to a tile he already ate, and he stops when the sum of the numbers on all eaten tiles is a multiple of nine. Find the minimum number of tiles that Hungryman eats.

S	7	9	16	18
25	27	36	45	52
54	63	70	72	81
88	90	99	108	115
117	124	126	133	135

Proposed by Kevin A. Zhou

Solution. $\boxed{18}$

Converting the table mod 9, we get:

S	7	0	7	0
7	0	0	0	7
0	0	7	0	0
7	0	0	0	7
0	7	0	7	0

Hungryman must go through all of the numbers that are $7 \pmod 9$ in order to get a sum that is $0 \pmod 9$. The shortest possible path lets Hungryman eat $\boxed{18}$ numbers. □

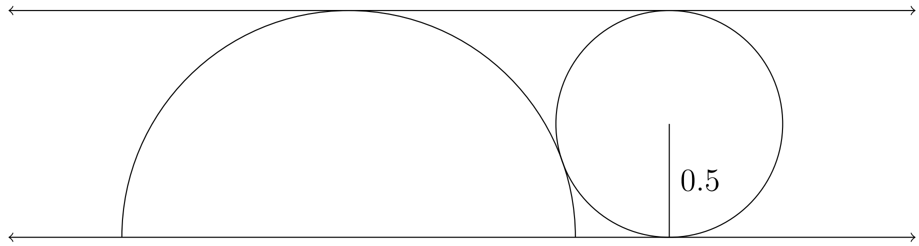
______ **15** **[5]** A semicircle of radius 1 has line ℓ along its base and is tangent to line m. Let r be the radius of the largest circle tangent to ℓ, m, and the semicircle. As the point of tangency on the semicircle varies, the range of possible values of r is the interval $[a, b]$. Find $b - a$.

Proposed by Steven Qu

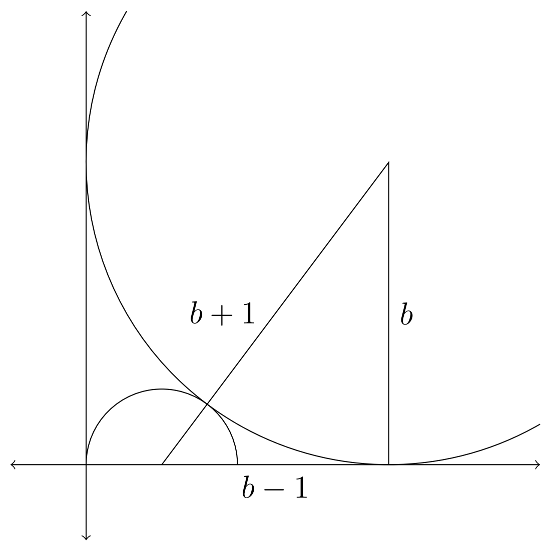
Solution. $\boxed{3.5}$

We only need the maximum and minimum values of r, which we determine to be when the point of tangency is at 0 or 90 degrees along the circle, respectively.

When m is parallel to ℓ, the radius is $a = 0.5$.



When m is perpendicular to ℓ, the radius of the circle can be found using the Pythagorean Theorem: $(b-1)^2 + b^2 = (b+1)^2$, which yields $b = 4$. Therefore, $b - a = \boxed{3.5}$.



□

MBMT Leibniz Guts Round — Set 4

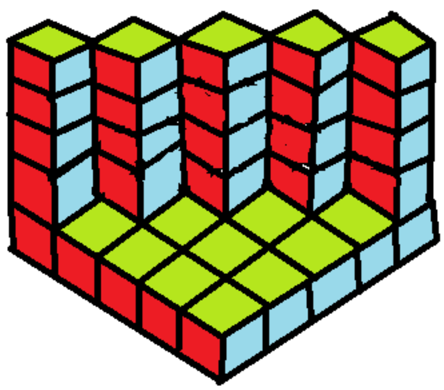
March 30, 2019

______ **16** **[7]** Anson, Billiam, and Connor are looking at a 3D figure. The figure is made of unit cubes and is sitting on the ground. No cubes are floating; in other words, each unit cube must either have another unit cube or the ground directly under it. Anson looks from the left side and says, "I see a 5×5 square." Billiam looks from the front and says the same thing. Connor looks from the top and says the same thing. Find the absolute difference between the minimum and maximum volume of the figure.

Proposed by Kevin A. Zhou

Solution. $\boxed{80}$

A $5 \times 5 \times 5$ cube maximizes the volume, so the maximum volume is $5^3 = 125$. To minimize the volume, let the figure be a $5 \times 5 \times 1$ prism resting on the ground, with a $1 \times 1 \times 4$ pillar protruding from each of 5 unit cubes such that no two pillars share the same row or column:



Thus, the minimum volume is $5^2 + 5 \cdot 4 = 45$. The absolute difference is $\boxed{80}$. □

______ **17** **[7]** The repeating decimal $0.\overline{MBMT}$ is equal to $\frac{p}{q}$, where p and q are relatively prime positive integers, and M, B, T are distinct digits. Find the minimum value of q.

Proposed by Kevin A. Zhou

Solution. $\boxed{303}$

If we let $x = 0.\overline{MBMT}$, then we can create the following equations:

$$10000x = MBMT.\overline{MBMT}$$
$$x = 0.\overline{MBMT}$$

Therefore, $9999x = \overline{MBMT}$, so $x = \frac{\overline{MBMT}}{9999}$. To minimize the denominator, we should try to introduce factors of 9999, namely $3^2, 11$, and 101, into the numerator.

Suppose $\overline{MBMT} = 101k$. Since $0 \le k \le 99$, notice that $101k$ is simply the number k, repeated. This would make the digits B and T equal, which is not allowed. Therefore, $101 \nmid \overline{MBMT}$.

Suppose $\overline{MBMT} = 99k$. This implies that $\overline{MBMT}$ is divisible by 9 and 11, and with our divisibility rules, we can write the following equations, where j, k are integers:

$$2M + B + T = 9j$$
$$2M - B - T = 11k$$

We must test out all values of j, k. To make our casework easier, first note that $B + T$ is positive, and therefore $9j > 11k$. Also, when we add the equations, we get $4M = 9j + 11k$, and thus $9j + 11k$ must be divisible by 4. The following cases should be examined:

- $j = 1, k = 1$. This implies $M = 5, B + T = -1$, clearly impossible.
- $j = 3, k = 1$. This implies $M = \frac{38}{4}$, so M can't be an integer, which is not allowed.
- $j = 3, k = -1$. This implies $M = 4, B + T = 19$, clearly impossible.
- $j = 4, k = 0$. This implies $M = 9, B + T = 18$; therefore, $B = 9, T = 9$, which is not allowed.

Therefore, $99 \nmid \overline{MBMT}$.

Suppose $\overline{MBMT} = 33k$. Now, our equations are:

$$2M + B + T = 3j$$
$$2M - B - T = 11k$$

A possible solution is $j = 4, k = 0, M = 3, B = 2, T = 4$. The denominator is $\frac{9999}{33} = \boxed{303}$. □

18 **[7]** Annie, Bob, and Claire have a bag containing the numbers $1, 2, 3, \ldots, 9$. Annie randomly chooses three numbers without replacement, then Bob chooses, then Claire gets the remaining three numbers. Find the probability that everyone is holding an arithmetic sequence. (Order does not matter, so 123, 213, and 321 all count as arithmetic sequences.)

Proposed by Haydn Gwyn

Solution. $\boxed{\frac{1}{56}}$

Ignoring which person gets each set of numbers, it is relatively easy to list out all possibilities. The possibilities are

$$123|456|789 \quad 123|468|579 \quad 135|246|789 \quad 147|258|369 \quad 159|234|678$$

We can order each of these five possibilities in six different ways, and there are $\binom{9}{3}\binom{6}{3} = 1680$ total ways for everyone to draw numbers, giving a final answer of $\frac{30}{1680} = \boxed{\frac{1}{56}}$. □

19 **[7]** Consider a set S of positive integers. Define the operation $f(S)$ to be the smallest integer $n > 1$ such that the base 2^k representation of n consists only of ones and zeros for all $k \in S$. Find the size of the largest set S such that $f(S) < 2^{2019}$.

Proposed by Haydn Gwyn

Solution. $\boxed{40}$

Consider all numbers in binary. If a number's base 2^k representation consists of only 0's and 1's, then the only digits in the binary representation that can be nonzero are the digits in the 2^{ik} place for positive integer i. For example, if a number's base 4 representation is all ones and zeros, its binary representation could be 1010101010101 or 10001 or 1000101. If a number's base 8 representation is all ones and zeros, its binary representation could be 1001001001001 or 1001000001. So we number the digits $0, 1, 2, \cdots$, where digit i is the 2^i place. If digit i can be nonzero, then all elements of S are factors of i. This means that the largest S with $f(S) < 2^{2019}$ is the set of factors of some number less than 2019.

Now we seek the number with the largest number of factors that is less than 2019. We start with 1. Each time we add a new prime to the factorization, the number of factors doubles. Each time we add a repeat prime to the factorization, the number of factors increases but does not double. So we take $210 = 2 \cdot 3 \cdot 5 \cdot 7$, as adding another prime puts us above 2019. We can multiply this by at most $\lfloor \frac{2019}{210} \rfloor = 9$ in order to keep the number below 2019. Multiplying by 8 is clearly the best option, so we have $2^4 \cdot 3 \cdot 5 \cdot 7 = 1680$. This number has $5 \cdot 2 \cdot 2 \cdot 2 = 40$ factors, so the answer to our question is 40. □

20 **[7]** Find the largest solution to the equation

$$2019(x^{2019x^{2019} - 2019^2 + 2019})^{2019} = 2019^{x^{2019}+1}.$$

Proposed by Steven Qu

Solution. $\boxed{2019^{\frac{1}{2019}}}$

Substitute $y = x^{2019}$. Then the equation becomes

$$y^{2019(y-2018)} = 2019^y.$$

Substituting $y = 2019^{a+1}$, this is

$$2019(a+1)(2019^{a+1} - 2018) = 2019^{a+1}$$

or

$$2019^a(2019a + 2018) = 2018(a+1).$$

This implies that

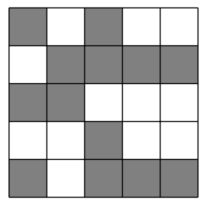
$$2019^{-a} = 1 + \frac{a}{2018(a+1)}.$$

Graphing both sides, it is now clear that the maximum value for a is 0. It follows that $x = \sqrt[2019]{2019}$. $\square$

MBMT Leibniz Guts Round — Set 5

March 30, 2019

______ **21** **[9]** Steven is concerned about his artistic abilities. To make himself feel better, he creates a 100×100 square grid and randomly paints each square either white or black, each with probability $\frac{1}{2}$. Then, he divides the white squares into connected components, groups of white squares that are connected to each other, possibly using corners. (For example, there are three connected components in the following diagram.) What is the expected number of connected components with 1 square, to the nearest integer?



Proposed by Daniel Zhu

Solution. $\boxed{25}$

The probability that each square in the middle is part of a 1 connected component is $\frac{1}{2^9}$, while for a square on the side it is $\frac{1}{2^6}$ and for a corner it is $\frac{1}{2^4}$. By linearity of expectation, we want

$$\frac{98^2}{512} + \frac{98 \cdot 4}{2^6} + \frac{4}{2^4} = \frac{2401}{128} + \frac{98}{16} + \frac{1}{4} = \frac{3217}{128} = 25 + \frac{17}{128}.$$

This rounds to $\boxed{25}$. □

______ **22** **[9]** Let x be chosen uniformly at random from $[0, 1]$. Let n be the smallest **positive** integer such that $3^n x$ is at most $\frac{1}{4}$ away from an integer. What is the expected value of n?

Proposed by Jacob Stavrianos

Solution. $\boxed{\frac{7}{4}}$

Let the answer be E. Take everything modulo 1.

The key realization is that if $3x \in [0, 1/3] \cup [2/3, 1]$, then n is either 1 or 2. In particular, given this case, there is a $3/4$ probability in landing in $[0, 1/4] \cup [3/4, 1]$ and having $n = 1$, and a $1/4$ probability of landing in $[1/4, 1/3] \cup [2/3, 3/4]$ and getting $n = 2$. So the expected value is $5/4$ in this case.

Consider the possiblity that $3x \in [1/3, 2/3]$. Then $n \neq 1$. Furthermore, since $9x$ is then distributed uniformly in $[0, 1]$, the expected value is simply $E + 1$.

Therefore

$$E = \frac{2}{3} \cdot \frac{5}{4} + \frac{E+1}{3} \implies E = \frac{5}{4} + \frac{1}{2} = \boxed{\frac{7}{4}}.$$

□

______ **23** **[9]** Let A and B be two points in the plane with $AB = 1$. Let ℓ be a variable line through A. Let ℓ' be a line through B perpendicular to ℓ. Let X be on ℓ and Y be on ℓ' with $AX = BY = 1$. Find the length of the locus of the midpoint of XY.

Proposed by Daniel Zhu

Solution. $\boxed{\sqrt{2}\pi}$

Let M be the midpoint of AB. Imagine shifting ℓ and ℓ' so that A and B are taken to M and letting this take X and Y to X' and Y'. Since X and Y were shifted in opposite directions, the midpoint of $X'Y'$ is the midpoint of XY. Also, since $MX' = MY' = 1$, it is clear that the midpoint of $X'Y'$ goes along a circle of radius $1/\sqrt{2}$ centered at M. The circumference of this circle is $\boxed{\sqrt{2}\pi}$. □

______ **24** **[9]** Each of the numbers a_i, where $1 \leq i \leq n$, is either -1 or 1. Also,

$$a_1a_2a_3a_4 + a_2a_3a_4a_5 + \cdots + a_{n-3}a_{n-2}a_{n-1}a_n + a_{n-2}a_{n-1}a_na_1 + a_{n-1}a_na_1a_2 + a_na_1a_2a_3 = 0.$$

Find the number of possible values for n between 4 and 100, inclusive.

Proposed by Daniel Monroe

Solution. $\boxed{24}$

Let S be the long expression set equal to zero.

Observe that if we change the sign of one of the a_i, we flip the sign of 4 terms in S. Each flip changes S by ± 2, so effecting an even number of flips ensures that S remains constant modulo 4. By setting all the a_i equal to 1, we can get $S = n$, so therefore $S \equiv n \pmod 4$ always. Therefore $4 \mid n$. Also 4 is clearly impossible, so this leaves us with 24 candidates. We will show that these all work.

Start with $a_1 = -1$ and set all the others equal to 1. One can show that $S = n - 8$. By assumption, this is a nonnegative multiple of 4. We will now decrease S in increments of 4 until it reaches 0.

Recall that the way to decrease S by 4 is to flip three positive terms and a negative term to three negative terms and a positive term, so we only need to find four adjacent terms, three of which are positive and one of which is negative. To do this, consider a "sliding window" going around the circle of terms. The number of negative terms can change by at most one, so if the desired arrangement does not exist, then there must always be zero negative terms or there must always be two or more. The first case is impossible since then S would be n, which contradicts our assumption that we would be decreasing S. The second case is also impossible since it would imply (by summing all the windows), that at least half the terms would be negative, leading to $S \leq 0$. □

______________ **25** **[9]** Let S be the set of positive integers less than 3^{2019} that have only zeros and ones in their base 3 representation. Find the sum of the squares of the elements of S. Express your answer in the form $a^b(c^d-1)(e^f-1)$, where a,b,c,d,e,f are positive integers and a,c,e are not perfect powers.

Proposed by Daniel Zhu

Solution. $\boxed{2^{2014}(3^{2019}-1)(3^{2020}-1)}$

We first note that we can instead find the sum of nonnegative integers less than 3^{2019} that only contain 0 and 1 in base 3. Consider a random base 3 number satisfying these conditions. We can write it as $n = a_{2018}\dots a_0$ in base 3 where a_i are either 1 or 0. We have

$$n^2 = \left(\sum_{i=0}^{2018} 3^i a_i\right)^2 = \sum_{i=0}^{2018} 3^{2i}a_i^2 + 2\sum_{0\le i<j\le 2018} 3^{i+j}a_ia_j.$$

We can compute the expected value of this using linearity of expectation. We first compute the expected value of the first sum. The expected value of $a_i^2 = \frac{1}{2}$. Thus the expected value of the first sum is

$$\frac{1}{2}\sum_{i=0}^{2018} 3^{2i} = \frac{9^{2019}-1}{16}.$$

Now we compute the expected value of the second sum. The expected value of a_ia_j is equal to $\frac{3}{4}\cdot 0 + \frac{1}{4}\cdot 1 = \frac{1}{4}$. Thus the expected value of the second sum is

$$\begin{aligned}\frac{1}{4}\cdot 2\sum_{0\le i<j\le 2018} 3^{i+j} &= \frac{1}{4}\left(\sum_{i=0}^{2018} 3^i\right)^2 - \frac{1}{4}\sum_{i=1}^{2018} 3^{2i} \\ &= \frac{9^{2019}-2\cdot 3^{2019}+1}{16} - \frac{9^{2019}-1}{32} = \frac{9^{2019}-4\cdot 3^{2019}+3}{32}.\end{aligned}$$

The expected value is the sum, which is

$$\frac{(3^{2019}-1)(3^{2020}-1)}{32}$$

by factoring a $3^{2019}-1$ out of each sum and simplifying. The sum is just the number of elements multiplied by this. The number of elements is 2^{2019}, so our answer is $2^{2014}(3^{2019}-1)(3^{2020}-1)$. □

MBMT Leibniz Guts Round — Set 6

March 30, 2019

This round is an estimation round. No one is expected to get an exact answer to any of these questions, but unlike other rounds, you will get points for being close. In the interest of transparency, the formulas for determining the number of points you will receive are located on the answer sheet, but they aren't very important when solving these problems.

To receive points, your answers should be positive and in decimal notation. For example, 10.55 is allowed, but not -3.2 or $\frac{2\pi}{3}$.

__________ **26** **[12]** What is the sum over all MBMT volunteers of the number of times that volunteer has attended MBMT (as a contestant or as a volunteer, including this year)?

Last year there were 47 volunteers; this is the fifth MBMT.

Proposed by Steven Qu

Solution. $\boxed{119}$

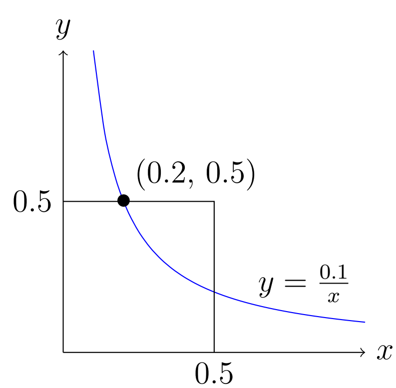
□

__________ **27** **[12]** William is sharing a chocolate bar with Naveen and Kevin. He first randomly picks a point along the bar and splits the bar at that point. He then takes the smaller piece, randomly picks a point along it, splits the piece at that point, and gives the smaller resulting piece to Kevin. Estimate the probability that Kevin gets less than 10% of the entire chocolate bar.

Proposed by Kevin A. Zhou

Solution. $\boxed{0.76651629275}$

Another way to think of this problem is: William chooses 2 random numbers $0 < x, y < 0.5$. Then, the smallest piece has size xy. We want to find the probability that $xy < 0.1$. We can think of this as throwing a dart into a square with side length 0.5, as shown below:



The area in the square that is under the blue curve is:

$$0.1 + \int_{0.2}^{0.5} \frac{0.1}{x} dx$$

Meanwhile, the total area of the square is 0.25. Some computation later, we get that the probability is $\boxed{0.7665}$. □

____________ **28** **[12]** Let x be the positive solution to the equation $x^{x^{x^x}} = 1.1$. Estimate $\frac{1}{x-1}$.

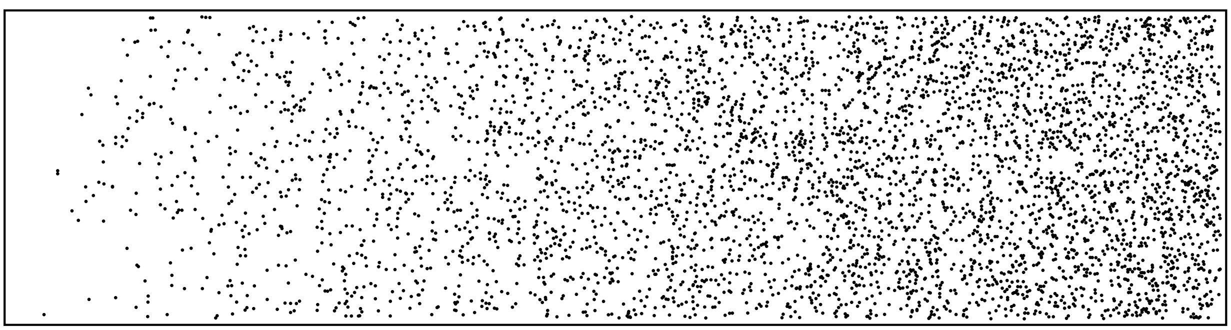
Proposed by Jacob Stavrianos

Solution. $\boxed{11.047666475590631}$

Notice that the sequence $x, x^x, x^{x^x}, \ldots$ is bounded given that x is "sufficiently small". Specifically, if we have $x^c = c$ for some positive c, then clearly $x < c$, and thus $x^x < x^c = c$, and so on. Thus, the sequence $x, x^x, x^{x^x} \ldots$ converges (quickly) to $x^{x^{x^{\cdots}}} = c \to x^c = c$.

Thus, we can instead solve $x^{1.1} = 1.1$. Applying Bernoulli's inequality, we get $x = 1.1^{\frac{1}{1.1}} \approx 1 + (.1)\frac{1}{1.1} = \frac{12}{11}$. Our approximation is then $\frac{1}{x-1} \approx 11$, which is close enough for nearly full points. □

____________ **29** **[12]** Estimate the number of dots in the following box:



It may be useful to know that this image was produced by plotting $(4\sqrt{x}, y)$ some number of times, where x, y are random numbers chosen uniformly randomly and independently from the interval $[0, 1]$.

Proposed by Daniel Zhu

Solution. $\boxed{4670}$

□

____________ **30** **[12]** For a positive integer n, let $f(n)$ be the smallest prime greater than or equal to n. Estimate

$$(f(1)-1)+(f(2)-2)+(f(3)-3)+\cdots+(f(10000)-10000).$$

Proposed by Haydn Gwyn

Solution. $\boxed{57134}$

□