

# MBMT Geometry Round – Leibniz

March 30, 2019

Full Name \_\_\_\_\_

Team Number \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- \_\_\_\_\_ 1 Let  $ABCDEF$  be a regular hexagon. Given that  $AD = 5$ , find  $AB$ .
- \_\_\_\_\_ 2 Circles  $A$ ,  $B$ , and  $C$  are all externally tangent, with radii 1, 10, and 100, respectively. What is the radius of the smallest circle entirely containing all three circles?
- \_\_\_\_\_ 3 The hour hand of a clock is 6 inches long, and the minute hand is 10 inches long. Find the area of the region swept out by the hands from 8:45AM to 9:15AM of a single day, in square inches.
- \_\_\_\_\_ 4 In rectangle  $ABCD$ ,  $AB = 2$  and  $AD > AB$ . Two quarter circles are drawn inside of  $ABCD$  with centers at  $A$  and  $C$  that pass through  $B$  and  $D$ , respectively. If these two quarter circles are tangent, find the area inside of  $ABCD$  that is outside both of the quarter circles.
- \_\_\_\_\_ 5 Find the area of a triangle with side lengths  $\sqrt{2}$ ,  $\sqrt{58}$ , and  $2\sqrt{17}$ .
- \_\_\_\_\_ 6 Triangle  $ABC$  is equilateral. A circle passes through  $A$  and is tangent to side  $BC$ . It intersects sides  $AB$  and  $AC$  again at  $E$  and  $F$ , respectively. If  $AE = 10$  and  $AF = 11$ , find  $AB$ .
- \_\_\_\_\_ 7 Triangle  $ABC$  has area 80. Point  $D$  is in the **interior** of  $\triangle ABC$  such that  $AD = 6$ ,  $BD = 4$ ,  $CD = 16$ , and the area of  $\triangle ADC = 48$ . Determine the area of  $\triangle ADB$ .
- \_\_\_\_\_ 8 Given two points  $A$  and  $B$  in the plane with  $AB = 1$ , define  $f(C)$  to be the circumcenter of triangle  $ABC$ , if it exists. Find the number of points  $X$  so that  $f^{2019}(X) = X$ .