MBMT Counting and Probability Round — Leibniz

March 30, 2019

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

____________ **1** An ant is at one corner of a cube of side length 1 and can move only along the edges of the cube. How many paths of length 3 can the ant take to the opposite corner of the cube?

____________ **2** Felix the Frog is in the middle of an endless staircase. On every hop, he can either hop 9 steps down or 5 steps up. Felix hops 100 times. At how many possible locations can Felix end his hopping route?

____________ **3** Two points are randomly selected inside a rectangle. What is the probability that the segment connecting these two points crosses at least one of the rectangle's diagonals?

____________ **4** In a class of 4 students, everyone is friends with each other. (No one is friends with themselves, so everyone has 3 friends.) How many ways are there to break at least one of these friendships so that everyone still has an odd number of friends?

____________ **5** Given a regular tetrahedron, how many ways are there to color two edges red, two edges green, and two edges blue? Rotations and reflections of a configuration are considered the same configuration.

Note: A regular tetrahedron is a triangular pyramid with all faces equilateral triangles.

____________ **6** Mr. Anderson rolls n fair six-sided dice, where $n \in \{1, 2, 3, 4, 5, 6\}$ is chosen uniformly at random. Given that the sum of the dice rolls is 6, find the probability that $n = 3$.

____________ **7** Steven starts with the number 1. Then, he repeats the following procedure N times: if he has the number n, he adds a random integer from 1 to $\gcd(n, 4)$, inclusive, to n. If $N = 2019^{2019^{2019}}$, find the closest integer to $100p$, where p is the probability that Steven's final number is divisible by 4.

____________ **8** There are 4 traffic lights placed uniformly at random on a 4-mile road. The traffic lights, all in sync, follow a 1-minute loop where they are red for 1 minute and then momentarily flash green so that only cars already stopped at the light can go on. A car arrives at one end of the road just as the lights are flashing green. If the car, when not stopped, always travels at 1 mile per minute, what is the expected number of minutes it takes for the car to traverse the road?