MBMT Geometry Round — Descartes

March 30, 2019

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

1 Triangle ABC has $AB = 3$, $BC = 4$, and $\angle B = 90°$. Find the area of triangle ABC.

Proposed by Jacob Stavrianos

Solution. $\boxed{6}$

The base is 3 and the height is 4, so the area is $\frac{1}{2}(3 \cdot 4) = \boxed{6}$ □

2 Let $ABCDEF$ be a regular hexagon. Given that $AD = 5$, find AB.

Proposed by Daniel Zhu

Solution. $\boxed{2.5}$

Dissect the hexagon into six equilateral triangles. Then it is clear that AD is twice the side length, so $AB = \boxed{\frac{5}{2}}$. □

3 Caroline glues two pentagonal pyramids to the top and bottom of a pentagonal prism so that the pentagonal faces coincide. How many edges does Caroline's figure have?

Proposed by Daniel Zhu

Solution. $\boxed{25}$

Each pyramid adds 5 edges to the 15 of the prism. So there are $15 + 2 \cdot 5 = \boxed{25}$ in total. □

4 The hour hand of a clock is 6 inches long, and the minute hand is 10 inches long. Find the area of the region swept out by the hands from 8:45AM to 9:15AM of a single day, in square inches.

Proposed by Daniel Zhu

Solution. $\boxed{\dfrac{203\pi}{4}}$

The minute hand travels 180 degrees, so it sweeps out $\frac{100\pi}{2}$. The hour hand travels 15 degrees, but it overlaps with the minute hand half the time. So, it sweeps out $\frac{36\pi}{48}$. The total is $\boxed{\frac{203\pi}{4}}$. □

5 Circles A, B, and C are all externally tangent, with radii 1, 10, and 100, respectively. What is the radius of the smallest circle entirely containing all three circles?

Proposed by Jacob Stavrianos

Solution. $\boxed{110}$

Consider the line segment that goes through the centers of B, C, and stops at the ends of the circles. This segment has length $200 + 20 = 220$, and since it must be entirely contained within the circle, the answer is at least 110. Since it is the diameter of a circle that contains all three circles, the radius is $\boxed{110}$. □

______ **6** Four parallel lines are drawn such that they are equally spaced and pass through the four vertices of a unit square. Find the distance between any two consecutive lines.

Proposed by Haydn Gwyn

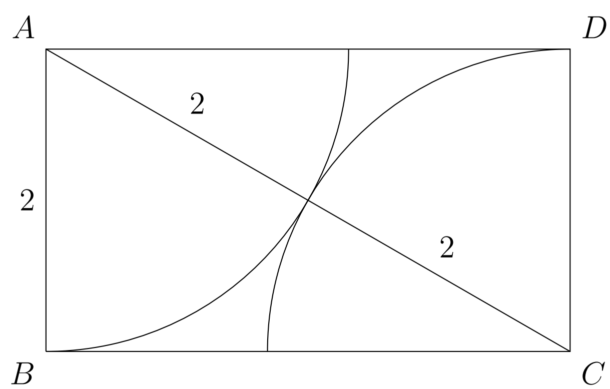
Solution. $\boxed{\dfrac{1}{\sqrt{5}}}$

First, we orient the square so that the four parallel lines are vertical. Let the distance between each pair of consecutive vertical lines be x. Note that the horizontal and vertical distances between two consecutive vertices of the square are x and $2x$ (not necessarily respectively), so we get the equation $x^2 + 4x^2 = 5x^2 = 1 \implies x = \boxed{\frac{1}{\sqrt{5}}}$. □

______ **7** In rectangle $ABCD$, $AB = 2$ and $AD > AB$. Two quarter circles are drawn inside of $ABCD$ with centers at A and C that pass through B and D, respectively. If these two quarter circles are tangent, find the area inside of $ABCD$ that is outside both of the quarter circles.

Proposed by Haydn Gwyn

Solution. $\boxed{4\sqrt{3} - 2\pi}$



Draw segment AC. Observe that, by symmetry, the point of tangency of the two quarter circles is the midpoint of this segment, so the length of AC is $2 + 2 = 4$ (note that the radius of both circles is 2, as AB and CD are radii of the quarter circles). Then we have $AC = \sqrt{4^2 - 2^2} = 2\sqrt{3}$. The area of the rectangle is then $2 \cdot 2\sqrt{3} = 4\sqrt{3}$, and the area of both of the quarter circles is $\frac{2}{4} \cdot \pi(2^2) = 2\pi$. Thus, the area inside of the rectangle but outside of both quarter circles is $4\sqrt{3} - 2\pi$. □

______ **8** Triangle ABC is equilateral. A circle passes through A and is tangent to side BC. It intersects sides AB and AC again at E and F, respectively. If $AE = 10$ and $AF = 11$, find AB.

Proposed by Daniel Zhu

Solution. $\boxed{7+\frac{2\sqrt{111}}{3}}$

Let x be the side-length. By using Power of a Point on B and C we get

$$x = \sqrt{x(x-10)} + \sqrt{x(x-11)}.$$

The rest is computation. □