

MBMT Counting and Probability Round – Descartes

March 30, 2019

Full Name _____

Team Number _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ 1 Kevin Zhou rolls two six-sided dice and writes down the sum of the two numbers shown. How many numbers could Kevin have written down?

_____ 2 An ant is at one corner of a cube of side length 1 and can move only along the edges of the cube. How many paths of length 3 can the ant take to the opposite corner of the cube?

_____ 3 What is the probability that a given two-digit multiple of 7 has a digit sum divisible by 7?

_____ 4 Felix the Frog is in the middle of an endless staircase. On every hop, he can either hop 9 steps down or 5 steps up. Felix hops 100 times. At how many possible locations can Felix end his hopping route?

_____ 5 Two points are randomly selected inside a rectangle. What is the probability that the segment connecting these two points crosses at least one of the rectangle's diagonals?

_____ 6 In a class of 4 students, everyone is friends with each other. (No one is friends with themselves, so everyone has 3 friends.) How many ways are there to break at least one of these friendships so that everyone still has an odd number of friends?

_____ 7 Given a regular tetrahedron, how many ways are there to color two edges red, two edges green, and two edges blue? Rotations and reflections of a configuration are considered the same configuration.

Note: A regular tetrahedron is a triangular pyramid with all faces equilateral triangles.

_____ 8 Steven starts with the number 1. Then, he repeats the following procedure N times: if he has the number n , he adds a random integer from 1 to $\gcd(n, 4)$, inclusive, to n . If $N = 2019^{2019^{2019}}$, find the closest integer to $100p$, where p is the probability that Steven's final number is divisible by 4.