MBMT Team Round — Gauss

April 7, 2018

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

DO NOT TURN THE QUESTION SHEET IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

____________ **1** Brandon wants to maximize $\frac{\square}{\square} + \square$ by placing the numbers 1, 2, and 3 in the boxes. If each number may only be used once, what is the maximum value attainable?

____________ **2** The planet Vriky is a sphere with radius 50 meters. Kyerk starts at the North Pole, walks straight along the surface of the sphere towards the equator, runs one full circle around the equator, and returns to the North Pole. How many meters did Kyerk travel in total throughout his journey?

____________ **3** Mr. Pham is lazy and decides Stan's quarter grade by randomly choosing an integer from 0 to 100 inclusive. However, according to school policy, if the quarter grade is less than or equal to 50, then it is bumped up to 50. What is the probability that Stan's final quarter grade is 50?

____________ **4** We define a function f such that for all integers n, k, x, we have that

$$f(n, kx) = k^n f(n, x) \text{ and } f(n+1, x) = xf(n, x)$$

If $f(1, k) = 2k$ for all integers k, then what is $f(3, 7)$?

____________ **5** What is the maximum (finite) number of points of intersection between the boundaries of a equilateral triangle of side length 1 and a square of side length 20?

____________ **6** An ant is on a coordinate plane. It starts at $(0, 0)$ and takes one step each second in the North, South, East, or West direction. After 5 steps, what is the probability that the ant is at the point $(2, 1)$?

____________ **7** Find a possible solution (B, E, T) to the equation $THE + MBMT = 2018$, where T, H, E, M, B represent distinct digits from 0 to 9.

____________ **8** A sequence of positive integers is constructed such that each term is greater than the previous term, no term is a multiple of another term, and no digit is repeated in the entire sequence. An example of such a sequence would be 4, 79, 1035. How long is the longest possible sequence that satisfies these rules?

____________ **9** A circle is tangent to two sides of an equilateral triangle of side length 1 as well as that same triangle's circumcircle. Find the radius of this circle.

____________ **10** Find the set of real numbers S so that

$$\prod_{c \in S}(x^2 + cxy + y^2) = (x^2 - y^2)(x^{12} - y^{12}).$$

____________ **11** ABC is an equilateral triangle of side length 8. P is a point on side AB. If $AC + CP = 5 \cdot AP$, find AP.

____________ **12** Given a function $f(x)$ such that $f(a+b) = f(a) + f(b) + 2ab$ and $f(3) = 0$, find $f(\frac{1}{2})$.

____________ **13** Badville is a city on the infinite Cartesian plane. It has 24 roads emanating from the origin, with an angle of 15 degrees between each road. It also has beltways, which are circles centered at the origin with any integer radius. There are no other roads in Badville. Steven wants to get from $(10, 0)$ to $(3, 3)$. What is the minimum distance he can take, only going on roads?

____________ **14** Team A and Team B are playing basketball. Team A starts with the ball, and the ball alternates between the two teams. When a team has the ball, they have a 50% chance of scoring 1 point. Regardless of whether or not they score, the ball is given to the other team after they attempt to score. What is the probability that Team A will score 5 points before Team B scores any?

____________ **15** The twelve-digit integer

$$\overline{A58B3602C91D},$$

where A, B, C, D are digits with $A > 0$, is divisible by 10101. Find $\overline{ABCD}$.