Team Number _____

# MBMT Gauss Guts Round — Set 1

**April 7, 2018**

___________ **1** **[3]** Daniel is exactly one year younger than his friend David. If David was born in the year 2008, in what year was Daniel born?

*Proposed by Dilhan Salgado*

*Solution.* $\boxed{2009}$

One year younger means born one year later, so the answer is $\boxed{2009}$. □

___________ **2** **[3]** John has a sheet of white paper which is 3 cm in height and 4 cm in width. He wants to paint the sky blue and the ground green so the entire paper is painted. If the ground takes up a third of the page, how much space (in $\text{cm}^2$) does the sky take up?

*Proposed by Kevin Qian*

*Solution.* $\boxed{8}$

The total area of the paper is $12\ \text{cm}^2$. Since a third is the ground and the rest is the sky, two-thirds of the paper is the sky, so the sky takes up $\frac{2}{3} \cdot 12 = \boxed{8\ \text{cm}^2}$. □

___________ **3** **[3]** Mr. Pham flips three coins. What is the probability that no two coins show the same side?

*Proposed by Daniel Zhu*

*Solution.* $\boxed{0}$

This is obviously impossible, so there is a $\boxed{0}$ chance of this occurring. □

___________ **4** **[3]** Find the last digit of

$$1333337777 \cdot 209347802 \cdot 3940704 \cdot 2309476091.$$

*Proposed by Daniel Zhu*

*Solution.* $\boxed{6}$

This is the last digit of $7 \cdot 2 \cdot 4 \cdot 1 = 56$, which is $\boxed{6}$. □

___________ **5** **[3]** Jihang and Eric are busy fidget spinning. While Jihang spins his fidget spinner at 15 revolutions per second, Eric only manages 10 revolutions per second. How many total revolutions will the two have made after 5 continuous seconds of spinning?

*Proposed by Anonymous*

*Solution.* $\boxed{125}$

They manage $15 + 10 = 25$ revolutions per second, so there are $25 \cdot 5 = \boxed{125}$ total revolutions. □

Team Number ______

# MBMT Gauss Guts Round — Set 2

**April 7, 2018**

______ **6** **[4]** Let $a$, $b$, and $c$ be real numbers. If $a^3 + b^3 + c^3 = 64$ and $a + b = 0$, what is the value of $c$?

*Proposed by Jyotsna Rao*

*Solution.* $\boxed{4}$

Rearranging the second equation, we find that $b = -a$. Substituting this value for $b$ into the first equation, we see that $a^3 - a^3 + c^3 = 64$. This means that $c^3 = 64$ and that $c = \boxed{4}$. □

______ **7** **[4]** Bender always turns 60 degrees clockwise. He walks 3 meters, turns, walks 2 meters, turns, walks 1 meter, turns, walks 4 meters, turns, walks 1 meter, and turns. How many meters does Bender have to walk to get back to his original position?

*Proposed by Guang Cui*

*Solution.* $\boxed{2}$

One way to do it is use a grid with equilateral triangles and trace Bender's path. Another way is that there is an equiangular hexagon with side lengths 3, 2, 1, 4, 1, and $x$ (in that order). Extending the sides and solving for $x$ gives $\boxed{2}$. □

______ **8** **[4]** You can buy a single piece of chocolate for 60 cents. You can also buy a packet with two pieces of chocolate for \$1.00. Additionally, if you buy four single pieces of chocolate, the fifth one is free. What is the lowest amount of money you have to pay for 44 pieces of chocolate? Express your answer in dollars and cents (ex. \$3.70).

*Proposed by Jyotsna Rao*

*Solution.* $\boxed{\$21.20}$

A group of five single pieces of chocolate costs $4 \cdot 60 = \$2.40$, so the first 40 pieces of chocolate cost $8 \cdot 2.40 = \$19.20$. The next four pieces of chocolate cannot be bought in a group of five, so it is cheapest to buy them in two packs of two. This costs \$2.00, so the total cost is $\boxed{\$21.20}$. □

______ **9** **[4]** Ten teams face off in a swim meet. The boys teams and girls teams are ranked independently, each team receiving some number of positive integer points, and the final results are obtained by adding the points for the boys and the points for the girls. If Blair's boys got 7th place while the girls got 5th place (no ties), what is the best possible total rank for Blair?

*Proposed by Guang Cui*

*Solution.* $\boxed{\text{2nd}}$

Blair could have beaten the teams that got 8th, 9th, and 10th place for boys as well as 6th, 7th, 8th, 9th, and 10th for girls, which could be 8 different teams in the best case. (Possibility: 1st place gets 1000, 2nd 999, ..., Blair 994, 8th place 3, 9th place 2, 10th place 1, etc.) However, there must be (at least) one team that beat Blair in both boys and girls, so the best possible is $\boxed{\text{2nd}}$ place. □

______________ **10** **[4]** On the planet Alletas, $\frac{32}{33}$ of the people with silver hair have purple eyes and $\frac{8}{11}$ of the people with purple eyes have silver hair. On Alletas, what is the ratio of the number of people with purple eyes to the number of people with silver hair?

*Proposed by Kevin Qian*

*Solution.* $\boxed{\frac{4}{3}}$

Let A = have purple eyes and B = have silver hair. Then

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{32}{33}$$
$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{8}{11}$$

Dividing the equations gives $\frac{P(A)}{P(B)} = \boxed{\frac{4}{3}}$ □

Team Number _____

# MBMT Gauss Guts Round — Set 3

**April 7, 2018**

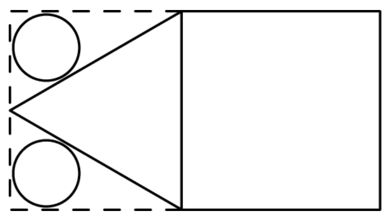
__________ **11** **[5]** Arlene has a square of side length 1, an equilateral triangle with side length 1, and two circles with radius 1/6. She wants to pack her four shapes in a rectangle without items piling on top of each other. What is the minimum possible area of the rectangle?

*Proposed by Guang Cui*

*Solution.* $\boxed{1 + \frac{\sqrt{3}}{2}}$

The square will account for at least 1 (with no space left), while the triangle will have to use up at least $\frac{\sqrt{3}}{2}$ area (since it is inscribed in some box, and the smallest box around an equilateral triangle has area $\frac{\sqrt{3}}{2}$), so this is optimal.

The construction takes a square and a triangle on top of it, with the two circles in the spaces between the triangle and the edge of the box. To show why this works, the "empty spaces" are 30-60-90 triangles with hypotenuse 1, which has inradius > 1/6.



□

__________ **12** **[5]** For how many integers $k$ is there an integer solution $x$ to the linear equation $kx + 2 = 14$?

*Proposed by Ambrose Yang*

*Solution.* $\boxed{12}$

The equation $kx = 12$ must yield an integer solution $x$. Thus $k$ must be a positive or negative factor of 12. 12 has 12 positive and negative factors. Therefore $k$ can take on $\boxed{12}$ different integral values. □

__________ **13** **[5]** Guang has 4 identical packs of gummies, and each pack has a red, a blue, and a green gummy. He eats all the gummies so that he finishes one pack before going on to the next pack, but he never eats two gummies of the same color in a row. How many different ways can Guang eat the gummies?

*Proposed by Guang Cui*

*Solution.* $\boxed{384}$

There are 6 ways for the first pack (RGB, RBG, etc.) For the second (and remaining) pack, only 4 of the 6 are valid, since the first color can't be the last color of the first pack. So there are $6 \cdot 4 \cdot 4 \cdot 4 = \boxed{384}$ ways. $\square$

______________ **14** **[5]** The numbers 5 and 7 are written on a whiteboard. Every minute Stev replaces the two numbers on the board with their sum and difference. After 2017 minutes the product of the numbers on the board is $m$. Find the number of factors of $m$.

*Proposed by Dilhan Saigado*

*Solution.* $\boxed{4040}$

Note that if we have $a, b$ on the board now, in next two minutes we will transition to $a+b, a-b$ and then $2a, 2b$, so the numbers doubles every 2 minutes. After 1 minute we have $2, 12$ on the board, so after 2017 we have $2 \cdot 2^{1008}, 12 \cdot 2^{1008}$ on the board. Therefore, the product is $3 \cdot 2^{2019}$, which has $2 \cdot 2020 = \boxed{4040}$ factors $\square$

______________ **15** **[5]** Let $P$ be a point on $y = -1$. Let the clockwise rotation of $P$ by $60^\circ$ about $(0,0)$ be $P'$. Find the minimum possible distance between $P'$ and $(0,-1)$.

*Proposed by Kevin Qian*

*Solution.* $\boxed{\frac{1}{2}}$

Consider the rotation of the line $y = -1$ by $60^\circ$. Then we want the shortest distance from the rotated line to $(0,-1)$. The rotated line is $y = -x\sqrt{3} - 2$. The distance from this to $(0,-1)$ is

$$\frac{-1 + 0\sqrt{3} + 2}{\sqrt{1+3}} = \boxed{\frac{1}{2}}$$

$\square$

Team Number _____

# MBMT Gauss Guts Round — Set 4

**April 7, 2018**

___________ **16** **[7]** A number $k$ is the product of exactly three distinct primes (in other words, it is of the form $pqr$, where $p, q, r$ are distinct primes). If the average of its factors is 66, find $k$.

*Proposed by Pratik Rathore*

*Solution.* $\boxed{258}$

Let $k = pqr$, then the sum of the factors is $(p+1)(q+1)(r+1)$. $k$ must have $(1+1)(1+1)(1+1) = 2\cdot2\cdot2 = 8$ factors, so the average is $\frac{(p+1)(q+1)(r+1)}{8}$. This implies that $(p+1)(q+1)(r+1) = 528 = 2^4\cdot3\cdot11$.

Clearly one of $p+1, q+1, r+1$ is a multiple of 11. WLOG let $p+1$ be the multiple of 11. Then we can check values for p and find that the minimum is $p = 43$. This turns out to be the only possible value for p, since the minimum of $(q+1)(r+1)$ is $(2+1)(3+1) = 12$, so for $p > 43$, $(p+1)(q+1)(r+1) > 44\cdot12 = 528$. Thus $(q, r) = (2, 3)$ or $(3, 2)$. Either way, the value of $k$ is $43\cdot3\cdot2 = \boxed{258}$. □

___________ **17** **[7]** Find the number of lattice points contained on or within the graph of $\frac{x^2}{3} + \frac{y^2}{2} = 12$. Lattice points are coordinate points $(x, y)$ where $x$ and $y$ are integers.

*Proposed by Annie Zhao*

*Solution.* $\boxed{89}$

Rewrite the equation as $2x^2 + 3y^2 = 72$. By graphing the ellipse, the maximum integer coordinate of $x$ is 6 and the maximum integer coordinate of $y$ is 4. Look at the points in the first quadrant, excluding the axis. When $y = 1, 2x^2 = 69$ so $x \leq 5$. Similarly, when $y = 2, x \leq 5$, when $y = 3, x \leq 4$, when $y = 4, x \leq 3$. The total number of lattice points in the first quadrant is 17. By symmetry, the number of lattice points in each of the 4 quadrants is equal, so there are 68 lattice points. There are 12 lattice points on the $x$-axis and 8 lattice points on the $y$-axis, both excluding the origin. Therefore, the total number of lattice points is $68 + 12 + 8 + 1 = \boxed{89}$. □

___________ **18** **[7]** How many triangles can be made from the vertices and center of a regular hexagon? Two congruent triangles with different orientations are considered distinct.

*Proposed by Jyotsna Rao*

*Solution.* $\boxed{32}$

The hexagon has 6 vertices and 1 center, so we have 7 points from which to choose the 3 vertices of the triangle. There are $\binom{7}{3} = 35$ ways to do this. However, 3 of these "triangles" are actually just lines because they are formed from the center and opposite vertices of the hexagon. Thus, there are actually $35 - 3 = \boxed{32}$ different triangles. □

**19** **[7]** Cindy has a cone with height 15 inches and diameter 16 inches. She paints one-inch thick bands of paint in circles around the cone, alternating between red and blue bands, until the whole cone is covered with paint. If she starts from the bottom of the cone with a blue strip, what is the ratio of the area of the cone covered by red paint to the area of the cone covered by blue paint?

*Proposed by Jyotsna Rao*

*Solution.* $\boxed{\frac{8}{9}}$

The radius, height, and a line straight down the lateral surface of the cone all form a right triangle. We know that the radius of the cone is 8 and that the height is 15, so by the Pythagorean theorem, the lateral length of the cone is 17. This means that Cindy used 17 one-inch strips of paper to cover the cone. Now imagine that we cut a slit along the surface of the cone and then lay it out flat, as though we've unrolled the cone. The resulting 2D shape is a circle with one sector missing. Since we are looking for the ratio between the red and blue areas, we can pretend that we have a whole circle instead of part of a circle, because the whole circle has the same ratio. The resulting shape looks like 17 concentric circles. Let us calculate the area of the blue parts. We can do this by summing the area of each of the blue concentric circles, and subtracting out the area of the next smaller circle inside it. Doing so gives the following sum: $\pi(1^2 + (3^2 - 2^2) + (5^2 - 4)^2 + (7^2 - 6^2) + \ldots + (17^2 - 16^2))$. This can be rewritten as $\pi(1 + 5 + 9 + 13 + \ldots + 33)$. This is an arithmetic series with 9 terms, so we know that the sum is $9\pi \cdot \dfrac{1+33}{2} = 153\pi$. The total area of the big circle is $17^2\pi = 289\pi$, and everything which isn't blue is red, so the area of the red parts is $289\pi - 153\pi = 136\pi$. Thus, the ratio of red area to blue area is $\dfrac{136\pi}{153\pi} = \dfrac{8}{9}$. □

**20** **[7]** An even positive integer $n$ has an *odd factorization* if the largest odd divisor of $n$ is also the smallest odd divisor of $n$ greater than 1. Compute the number of even integers $n$ less than 50 with an *odd factorization*.

*Proposed by David Wu*

*Solution.* $\boxed{15}$

Let $n = 2^a \cdot b$, where $b$ is odd. Then $b$ is the largest odd divisor of $n$. If $b$ has no smaller odd divisors other than 1, then $b$ must be prime. We now perform casework on the value of $a$. Clearly $a < 6$. If $a = 5$, then $b = 3$ is already too big. If $a = 4$, then $b = 3$ is the only possibility. If $a = 3$, then $b = 3$ or 5. If $a = 2$, then $b = 3$, 5, 7, or 11. If $a = 1$, then $b = 3, 5, 7, 11, 13, 17, 19,$ or 23. Hence, the answer is $1 + 2 + 4 + 8 = \boxed{15}$. □

Team Number _____

# MBMT Gauss Guts Round — Set 5

**April 7, 2018**

___________ **21** **[9]** In the magical tree of numbers, $n$ is directly connected to $2n$ and $2n+1$ for all nonnegative integers $n$. A frog on the magical tree of numbers can move from a number $n$ to a number connected to it in 1 hop. What is the least number of hops that the frog can take to move from 1000 to 2018?

*Proposed by Jacob Stavrianos*

*Solution.* $\boxed{11}$

If we think of $n$ in binary, then going forward in the tree (to a bigger number) is appending a 0/1 digit to the end of the number and going backward in the tree (to a smaller number) is deleting a 0/1 digit from the end. We can now consider 1 hop as adding/deleting a binary digit from the end.

In binary:

$1000 = 1111101000$

$2018 = 11111100010$

Since the first 5 digits match, we don't need to change them. The remaining digits don't, so they must be changed. So to get from 1000 to 2018, we must do the following:

$1111101000 \to 11111$ by deleting 5 digits

$11111 \to 11111100010$ by appending 6 digits

For a total of $\boxed{11}$ hops. □

___________ **22** **[9]** Stan makes a deal with Jeff. Stan is given 1 dollar, and every day for 10 days he must either double his money or burn a perfect square amount of money. At first Stan thinks he has made an easy 1024 dollars, but then he learns the catch - after 10 days, the amount of money he has must be a multiple of 11 or he loses all his money. What is the largest amount of money Stan can have after the 10 days are up?

*Proposed by Dilhan Salgado*

*Solution.* $\boxed{462}$

Suppose that Stan doubles $i$ times and spends $10-i$ times. When $i = 10$, we can only get 1024, which is not a multiple of 11). When $i \leq 8$, we can get at most $256 - 1 - 1 = 254$. We will show we can do better if $i = 9$. Suppose that Stan doubles $9 - a$ times, spends $k^2$ dollars then doubles $a$ times for a total of $(2^{9-a} - k^2) * 2^a = 2^9 - 2^a k^2$ dollars. As $512 - 2^a k^2$ must be divisible by 11 and $512 \equiv 6 \pmod{11}$, we need to find the minimal $2^a k^2$ that is $6 \pmod{11}$. The possible values are $6, 17, 28, 39, 50, \ldots$. Before 50 we have an odd power of an odd prime, so those numbers aren't possible, but $50 = 2^1 \cdot 5^2$, so we can get $2^9 - 50 = 462$ dollar (specifically by doubling 8 times, spending 25 dollars then doubling one last time). □

______ **23** **[9]** Let $\Gamma_1$ be a circle with diameter 2 and center $O_1$ and let $\Gamma_2$ be a congruent circle centered at a point $O_2 \in \Gamma_1$. Suppose $\Gamma_1$ and $\Gamma_2$ intersect at $A$ and $B$. Let $\Omega$ be a circle centered at $A$ passing through $B$. Let $P$ be the intersection of $\Omega$ and $\Gamma_1$ other than $B$ and let $Q$ be the intersection of $\Omega$ and ray $\overrightarrow{AO_1}$. Define $R$ to be the intersection of $PQ$ with $\Gamma_1$. Compute the length of $O_2R$.

*Proposed by Kevin Qian*

*Solution.* $\boxed{\sqrt{2}}$

First, we claim $P, O_1, O_2$ are collinear. This is easy to see as $\widehat{AP} = \widehat{AB} = 120°$, so $\triangle PAB$ is in fact equilateral.

Observe that $\angle PQA = 0.5\angle PQB = 0.25\angle PAB = 15°$.

Let $QO_1$ intersect $\Gamma_1$ at $X$. Since

$$15°\angle RQA = 0.5(\widehat{PX} - \widehat{RQ}) = 0.5(60 - \widehat{RQ}),$$

we know $\widehat{RQ} = 30°$. Thus, $\angle RO_1O_2 = 90°$, and the answer is $\sqrt{2}$. □

______ **24** **[9]** 8 people are at a party. Each person gives one present to one other person such that everybody gets a present and no two people exchange presents with each other. How many ways is this possible?

*Proposed by Guang Cui*

*Solution.* $\boxed{8988}$

There are 3 possible ways for this to happen.

Case 1: Cycle of length 8; A to B, B to C, C to D, ..., H to A. There are 7! ways for this to happen.

Case 2: Cycle of length 5 and cycle of length 3; A to B, B to C, C to D, D to E, E to A, F to G, G to H, H to F. There are $\binom{8}{3} \cdot 4! \cdot 2!$ ways.

Case 3: Two cycles of length 4; A to B, B to C, C to D, D to A, E to F, F to G, G to H, H to E. There are $\frac{\binom{8}{4}}{2} \cdot 3! \cdot 3!$ ways.

In total, there are $7! + \binom{8}{3} \cdot 4! \cdot 2! + \frac{\binom{8}{4}}{2} \cdot 3! \cdot 3! = 8988$ total ways.

(Alternative methods such as PIE and complementary counting with derangements can be used as well.) $\square$

**25** **[9]** Let $S$ be the set of points $(x, y)$ such that $y = x^3 - 5x$ and $x = y^3 - 5y$. There exist four points in $S$ that are the vertices of a rectangle. Find the area of this rectangle.

*Proposed by Daniel Zhu*

*Solution.* $\boxed{2\sqrt{21}}$

First of all, graphing $S$ suggests that it contains 9 points. Due to the symmetries of $S$, there are four points that are the same distance from the origin, which form a rectangle. Further inspection of the symmetries suggests that the points are of the form $(a, b)$, $(b, a)$, $(-a, -b)$, and $(-b, -a)$ for some $a \neq \pm b$. By using the Shoelace formula or just geometry, we get that the area of the rectangle is $2|a^2 - b^2|$. So it suffices to just find $a$ and $b$.

Let $f(x) = x^3 - 5x$. Note that $x = f(x)$, then $(x, f(x)) \in S$. Also, since $f$ is an odd function, if $x = -f(x)$, then $(x, f(x)) \in S$ as well. Straightforward algebra yields that these solutions are $x = 0, \pm 2, \pm\sqrt{6}$. By the Fundamental Theorem of Algebra, there are 4 solutions left, which we need to find.

Now we expand

$$f(f(x)) = x^9 - 15x^7 + 75x^5 - 125x^3 - 5x^3 + 25x.$$

So

$$f(f(x)) - x = x^9 - 15x^7 + 75x^5 - 130x^3 + 24x.$$

We already know the roots $x = 0, \pm 2, \pm\sqrt{6}$, so we can factor out factors of $x$, $x^2 - 4$, and $x^2 - 6$ to get that at the other solutions for $x$ we have

$$x^4 - 5x^2 + 1 = 0.$$

This means that

$$\{a^2, b^2\} = \frac{5 \pm \sqrt{21}}{2},$$

so the area is $2\sqrt{21}$. $\square$

Team Number _____

# MBMT Gauss Guts Round — Set 6

**April 7, 2018**

**Note. Every answer in this section must be positive and given in decimal notation to receive points; 1000 and 4354.3 are allowed, while 2/3, $-34$, 0, and $\pi$ are not.**

**Also, some of the problems are on the back of this sheet.**

___________ **26** **[12]** When $2018! = 2018 \times 2017 \times \ldots \times 1$ is multiplied out and written as an integer, find the number of 4's.

If the correct answer is $A$ and your answer is $E$, you will receive $12\min(A/E, E/A)^3$ points.

*Proposed by Guang Cui*

*Solution.* $\boxed{538}$

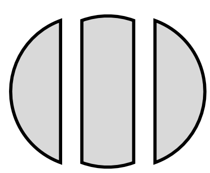
Note that the number of trailing 0's is $\lfloor \frac{2018}{5} \rfloor + \lfloor \frac{2018}{25} \rfloor + \ldots = 529$. Now, to compute the total number of digits in 2018! we can use Stirling's approximation or logs or other methods (or we may tell them how many digits there are, at least for division b). Turns out to be 5795 digits. $\frac{5795-529}{10} = 526.6$ is a good approximation, which assumes that digits are equally distributed besides the trailing zeroes.

The actual answer is $\boxed{538}$(!) and the actual distribution is

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1031 | 503 | 546 | 564 | 538 | 562 | 513 | 508 | 555 | 475 |

□

___________ **27** **[12]** A circle of radius 10 is cut into three pieces of equal area with two parallel cuts. Find the width of the center piece.



If the correct answer is $A$ and your answer is $E$, you will receive $\max(0, 12 - 6|A - E|)$ points.

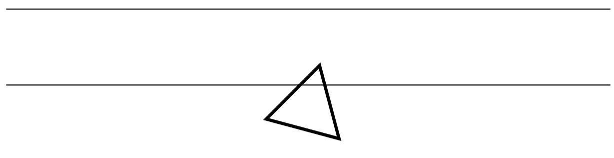
*Proposed by Guang Cui*

*Solution.* $\boxed{5.29864}$

The edge pieces should have area $\frac{100\pi}{3}$. If $\theta$ is the angle to the edge piece, then the area is equal to $100\pi \cdot \frac{\theta}{2\pi} - \frac{1}{2} \cdot 100 \cdot \sin\theta = \frac{100\pi}{3}$, so $\theta \approx 150°$.

The width is $20 \cdot \cos(\frac{\theta}{2})$, which is about $\boxed{5.299}$ □

_____________ **28** **[12]** An equilateral triangle of side length 1 is randomly thrown onto an infinite set of lines, spaced 1 apart.



On average, how many times will the boundary of the triangle intersect one of the lines? For example, in the above diagram, the boundary of the triangle intersects the lines in 2 places.

If the correct answer is $A$ and your answer is $E$, you will receive $\max(0, 12-120|A-E|/A)$ points.

*Proposed by Guang Cui*

*Solution.* $\boxed{\frac{6}{\pi}}$

This is a generalization of the problem where a stick of length one is thrown (the answer is $\frac{2}{\pi}$). Suppose the stick is thrown at angle $\theta$. Then in the band between two lines, the region where the center of the stick can be thrown and intersect some line is $\sin\theta$ of the band. Therefore, the probability is

$$\frac{\int_0^\pi \sin\theta \, d\theta}{\pi} = \frac{2}{\pi}$$

Now, by linearity of expectation, the answer is $3 \cdot \frac{2}{\pi} = \frac{6}{\pi}$.

Competitors are not expected to know this; rather they should reasonably see that it will usually intersect twice, and very rarely intersect 0 times. $\frac{6}{\pi}$ is about 1.9 □

_____________ **29** **[12]** Call an ordered triple of integers $(a, b, c)$ *nice* if there exists an integer $x$ such that $ax^2 + bx + c = 0$. How many *nice* triples are there such that $-100 \leq a, b, c \leq 100$?

If the correct answer is $A$ and your answer is $E$, you will receive $12 \min(A/E, E/A)$ points.

*Proposed by Daniel Zhu*

*Solution.* $\boxed{145319}$

Program it. You can use the rational root theorem to speed things up. □

____________ **30** **[12]** Let $f(i)$ denote the number of MBMT volunteers to be born in the $i$th state to join the United States. Find the value of $1f(1) + 2f(2) + 3f(3) + \cdots + 50f(50)$.
Note 1: Maryland was the 7th state to join the US.
Note 2: Last year's MBMT competition had 42 volunteers.

If the correct answer is $A$ and your answer is $E$, you will receive $\max(0, 12 - 500(|A - E|/A)^2)$ points.

*Proposed by Dilhan Salgado*

*Solution.* $\boxed{440}$

This year there were 47 MBMT volunteers, of which 41 were born in the United States. The breakdown was

- 12 born in Maryland
- 7 born in DC (which affects the answer by 0)
- 6 born in Pennsylvania
- 3 born in California and Texas
- 2 born in New Jersey and North Carolina
- 1 born in Ohio, Michigan, Wisconsin, New York, Illinois, and Minnesota

□