MBMT Gauss Guts Round — Set 1

April 7, 2018

_____ **1 [3]** Daniel is exactly one year younger than his friend David. If David was born in the year 2008, in what year was Daniel born?

_____ **2 [3]** John has a sheet of white paper which is 3 cm in height and 4 cm in width. He wants to paint the sky blue and the ground green so the entire paper is painted. If the ground takes up a third of the page, how much space (in cm^2) does the sky take up?

_____ **3 [3]** Mr. Pham flips three coins. What is the probability that no two coins show the same side?

_____ **4 [3]** Find the last digit of

$$1333337777 \cdot 209347802 \cdot 3940704 \cdot 2309476091.$$

_____ **5 [3]** Jihang and Eric are busy fidget spinning. While Jihang spins his fidget spinner at 15 revolutions per second, Eric only manages 10 revolutions per second. How many total revolutions will the two have made after 5 continuous seconds of spinning?

MBMT Gauss Guts Round — Set 2
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_____ **6 [4]** Let a, b, and c be real numbers. If $a^3 + b^3 + c^3 = 64$ and $a + b = 0$, what is the value of c?

_____ **7 [4]** Bender always turns 60 degrees clockwise. He walks 3 meters, turns, walks 2 meters, turns, walks 1 meter, turns, walks 4 meters, turns, walks 1 meter, and turns. How many meters does Bender have to walk to get back to his original position?

_____ **8 [4]** You can buy a single piece of chocolate for 60 cents. You can also buy a packet with two pieces of chocolate for $1.00. Additionally, if you buy four single pieces of chocolate, the fifth one is free. What is the lowest amount of money you have to pay for 44 pieces of chocolate? Express your answer in dollars and cents (ex. $3.70).

_____ **9 [4]** Ten teams face off in a swim meet. The boys teams and girls teams are ranked independently, each team receiving some number of positive integer points, and the final results are obtained by adding the points for the boys and the points for the girls. If Blair's boys got 7th place while the girls got 5th place (no ties), what is the best possible total rank for Blair?

_____ **10 [4]** On the planet Alletas, $\frac{32}{33}$ of the people with silver hair have purple eyes and $\frac{8}{11}$ of the people with purple eyes have silver hair. On Alletas, what is the ratio of the number of people with purple eyes to the number of people with silver hair?

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_____ **11 [5]** Arlene has a square of side length 1, an equilateral triangle with side length 1, and two circles with radius 1/6. She wants to pack her four shapes in a rectangle without items piling on top of each other. What is the minimum possible area of the rectangle?

_____ **12 [5]** For how many integers k is there an integer solution x to the linear equation $kx + 2 = 14$?

_____ **13 [5]** Guang has 4 identical packs of gummies, and each pack has a red, a blue, and a green gummy. He eats all the gummies so that he finishes one pack before going on to the next pack, but he never eats two gummies of the same color in a row. How many different ways can Guang eat the gummies?

_____ **14 [5]** The numbers 5 and 7 are written on a whiteboard. Every minute Stev replaces the two numbers on the board with their sum and difference. After 2017 minutes the product of the numbers on the board is m. Find the number of factors of m.

_____ **15 [5]** Let P be a point on $y = -1$. Let the clockwise rotation of P by $60°$ about $(0,0)$ be P'. Find the minimum possible distance between P' and $(0,-1)$.

MBMT Gauss Guts Round — Set 4
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_____ **16 [7]** A number k is the product of exactly three distinct primes (in other words, it is of the form pqr, where p, q, r are distinct primes). If the average of its factors is 66, find k.

_____ **17 [7]** Find the number of lattice points contained on or within the graph of $\frac{x^2}{3} + \frac{y^2}{2} = 12$. Lattice points are coordinate points (x, y) where x and y are integers.

_____ **18 [7]** How many triangles can be made from the vertices and center of a regular hexagon? Two congruent triangles with different orientations are considered distinct.

_____ **19 [7]** Cindy has a cone with height 15 inches and diameter 16 inches. She paints one-inch thick bands of paint in circles around the cone, alternating between red and blue bands, until the whole cone is covered with paint. If she starts from the bottom of the cone with a blue strip, what is the ratio of the area of the cone covered by red paint to the area of the cone covered by blue paint?

_____ **20 [7]** An even positive integer n has an *odd factorization* if the largest odd divisor of n is also the smallest odd divisor of n greater than 1. Compute the number of even integers n less than 50 with an *odd factorization*.

MBMT Gauss Guts Round — Set 5
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_____ **21** **[9]** In the magical tree of numbers, n is directly connected to $2n$ and $2n+1$ for all nonnegative integers n. A frog on the magical tree of numbers can move from a number n to a number connected to it in 1 hop. What is the least number of hops that the frog can take to move from 1000 to 2018?

_____ **22** **[9]** Stan makes a deal with Jeff. Stan is given 1 dollar, and every day for 10 days he must either double his money or burn a perfect square amount of money. At first Stan thinks he has made an easy 1024 dollars, but then he learns the catch - after 10 days, the amount of money he has must be a multiple of 11 or he loses all his money. What is the largest amount of money Stan can have after the 10 days are up?

_____ **23** **[9]** Let Γ_1 be a circle with diameter 2 and center O_1 and let Γ_2 be a congruent circle centered at a point $O_2 \in \Gamma_1$. Suppose Γ_1 and Γ_2 intersect at A and B. Let Ω be a circle centered at A passing through B. Let P be the intersection of Ω and Γ_1 other than B and let Q be the intersection of Ω and ray $\overrightarrow{AO_1}$. Define R to be the intersection of PQ with Γ_1. Compute the length of O_2R.

_____ **24** **[9]** 8 people are at a party. Each person gives one present to one other person such that everybody gets a present and no two people exchange presents with each other. How many ways is this possible?

_____ **25** **[9]** Let S be the set of points (x, y) such that $y = x^3 - 5x$ and $x = y^3 - 5y$. There exist four points in S that are the vertices of a rectangle. Find the area of this rectangle.

MBMT Gauss Guts Round — Set 6
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Note. Every answer in this section must be positive and given in decimal notation to receive points; 1000 and 4354.3 are allowed, while 2/3, -34, 0, and π are not.

Also, some of the problems are on the back of this sheet.

26 [12] When $2018! = 2018 \times 2017 \times \ldots \times 1$ is multiplied out and written as an integer, find the number of 4's.
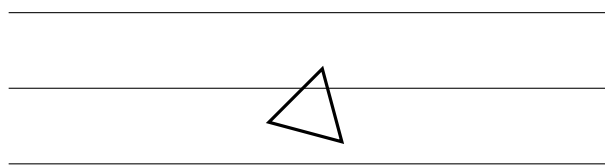
If the correct answer is A and your answer is E, you will receive $12 \min(A/E, E/A)^3$ points.

27 [12] A circle of radius 10 is cut into three pieces of equal area with two parallel cuts. Find the width of the center piece.
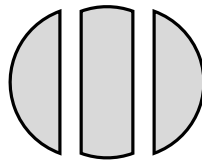
If the correct answer is A and your answer is E, you will receive $\max(0, 12 - 6|A - E|)$ points.

28 [12] An equilateral triangle of side length 1 is randomly thrown onto an infinite set of lines, spaced 1 apart.

On average, how many times will the boundary of the triangle intersect one of the lines? For example, in the above diagram, the boundary of the triangle intersects the lines in 2 places.

If the correct answer is A and your answer is E, you will receive $\max(0, 12 - 120|A - E|/A)$ points.

29 [12] Call an ordered triple of integers (a, b, c) *nice* if there exists an integer x such that $ax^2 + bx + c = 0$. How many *nice* triples are there such that $-100 \le a, b, c \le 100$?

If the correct answer is A and your answer is E, you will receive $12 \min(A/E, E/A)$ points.

30 **[12]** Let $f(i)$ denote the number of MBMT volunteers to be born in the ith state to join the United States. Find the value of $1f(1) + 2f(2) + 3f(3) + \cdots + 50f(50)$.

Note 1: Maryland was the 7th state to join the US.

Note 2: Last year's MBMT competition had 42 volunteers.

If the correct answer is A and your answer is E, you will receive $\max(0, 12 - 500(|A - E|/A)^2)$ points.