

MBMT Geometry Round – Gauss

April 7, 2018

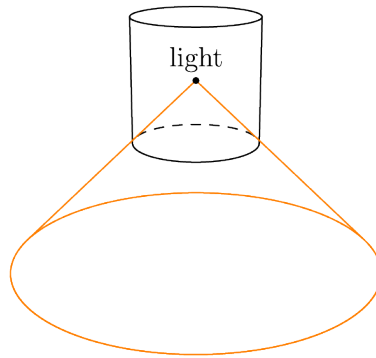
Full Name _____

Team Number _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- _____ 1 27 cubes of side length 1 are arranged to form a $3 \times 3 \times 3$ cube. If the corner $1 \times 1 \times 1$ cubes are removed, what fraction of the volume of the big cube is left?
- _____ 2 Points A , B , and C are on a line such that $AB = 6$ and $BC = 11$. Find all possible values of AC .
- _____ 3 Consider rectangle $ABCD$, with $1 = AB < BC$. The angle bisector of $\angle DAB$ intersects \overline{BC} at E and \overline{DC} at F . If $FE = FD$, find BC .
- _____ 4 Consider a lamp in the shape of a hollow cylinder with the circular faces removed with height 48 cm and radius 7 cm. A point source of light is situated at the center of the lamp. The lamp is held so that the bottom of the lamp is at a height 48 cm above an infinite flat sheet of paper. What is the area of the illuminated region on the flat sheet of paper, in cm^2 ?



- _____ 5 There exist two triangles ABC such that $AB = 13$, $BC = 12\sqrt{2}$, and $\angle C = 45^\circ$. Find the positive difference between their areas.
- _____ 6 $\triangle ABC$ is a right triangle with $\angle A = 90^\circ$. Square $ADEF$ is drawn, with D on \overline{AB} , F on \overline{AC} , and E inside $\triangle ABC$. Point G is chosen on \overline{BC} such that EG is perpendicular to BC . Additionally, $DE = EG$. Given that $\angle C = 20^\circ$, find the measure of $\angle BEG$.
- _____ 7 Let ABC be an equilateral triangle with side length 2. Let the circle with diameter AB be Γ . Consider the two tangents from C to Γ , and let the tangency point closer to A be D . Find the area of $\triangle CAD$.
- _____ 8 Let ABC be a triangle with $\angle A = 60^\circ$, $AB = 37$, $AC = 41$. Let H and O be the orthocenter and circumcenter of ABC , respectively. Find OH .

The orthocenter of a triangle is the intersection point of the three altitudes. The circumcenter of a triangle is the intersection point of the three perpendicular bisectors of the sides.