

MBMT Algebra Round — Gauss

April 7, 2018

Full Name _____

Team Number _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** UMBC and UVA are playing a basketball game. If UMBC scores 12 three-pointers, 14 two-pointers, and 10 free throws (worth one point), while UVA scores 4 three-pointers, 19 two-pointers, and 4 free throws, then by how much did UMBC beat UVA?

_____ **2** To test Bob's memory, Alice tells Bob the numbers 1 through 10 in some order, but she skips one number. Bob is supposed to, in return, tell Alice the skipped number. Bob doesn't have a great memory, but he is clever, so he sums up the numbers Alice tells him. If Bob gets a sum of 50, what is the missing number?

_____ **3** Guang's watch runs 1% slower than normal time. Luckily, he resets the time on his watch to be equal to the actual time at 6 AM, 11 AM, 4 PM, and 10 PM every day. What is the maximum difference in seconds ever achieved between the time on Guang's watch and the actual time?

_____ **4** If $a^2 + 2b^2 = 72$ and $(a + 2b)^2 = 144$, and neither a nor b is equal to 0, find ab .

_____ **5** Squares of side length x ($x < \frac{9}{2}$) are cut out of each corner of a 9 by 10 rectangular sheet of paper. The paper is then folded up into an open box. If the box has volume 60, find all possible values of x .

_____ **6** Find the minimum value of $(x - y + 1)^2 + (xy + y + 1)^2$ over all pairs of real numbers (x, y) .

_____ **7** Find the unique positive solution to $x[x[x]]\lceil x \rceil = 130$. Here, $\lfloor x \rfloor$ is the largest integer less than or equal to x , and $\lceil x \rceil$ is the smallest integer greater than or equal to x .

_____ **8** Find the minimum value of

$$a + \frac{8}{a} + \frac{8}{a + \frac{8}{a}}$$

where a is a positive real number.