MBMT Team Round — Cantor

April 7, 2018

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

DO NOT TURN THE QUESTION SHEET IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

_____ **1** Mr. Pham flips 2018 coins. What is the difference between the maximum and minimum number of heads that can appear?

_____ **2** Brandon wants to maximize $\frac{\square}{\square} + \square$ by placing the numbers 1, 2, and 3 in the boxes. If each number may only be used once, what is the maximum value attainable?

_____ **3** Guang has 10 cents consisting of pennies, nickels, and dimes. What are all the possible numbers of pennies he could have?

_____ **4** The ninth edition of Campbell Biology has 1464 pages. If Chris reads from the beginning of page 426 to the end of page 449, what fraction of the book has he read?

_____ **5** The planet Vriky is a sphere with radius 50 meters. Kyerk starts at the North Pole, walks straight along the surface of the sphere towards the equator, runs one full circle around the equator, and returns to the North Pole. How many meters did Kyerk travel in total throughout his journey?

_____ **6** Mr. Pham is lazy and decides Stan's quarter grade by randomly choosing an integer from 0 to 100 inclusive. However, according to school policy, if the quarter grade is less than or equal to 50, then it is bumped up to 50. What is the probability that Stan's final quarter grade is 50?

_____ **7** What is the maximum (finite) number of points of intersection between the boundaries of a equilateral triangle of side length 1 and a square of side length 20?

_____ **8** You enter the MBMT lottery, where contestants select three different integers from 1 to 5 (inclusive). The lottery randomly selects two winning numbers, and tickets that contain both of the winning numbers win. What is the probability that your ticket will win?

_____ **9** Find a possible solution (B, E, T) to the equation $THE + MBMT = 2018$, where T, H, E, M, B represent distinct digits from 0 to 9.

_____ **10** $ABCD$ is a unit square. Let E be the midpoint of AB and F be the midpoint of AD. DE and CF meet at G. Find the area of $\triangle EFG$.

_____ **11** The eight numbers 2015, 2016, 2017, 2018, 2019, 2020, 2021, and 2022 are split into four groups of two such that the two numbers in each pair differ by a power of 2. In how many different ways can this be done?

_____ **12** We define a function f such that for all integers n, k, x, we have that

$$f(n, kx) = k^n f(n, x) \text{ and } f(n+1, x) = xf(n, x)$$

If $f(1, k) = 2k$ for all integers k, then what is $f(3, 7)$?

_____ **13** A sequence of positive integers is constructed such that each term is greater than the previous term, no term is a multiple of another term, and no digit is repeated in the entire sequence. An example of such a sequence would be 4, 79, 1035. How long is the longest possible sequence that satisfies these rules?

_____ **14** ABC is an equilateral triangle of side length 8. P is a point on side AB. If $AC + CP = 5 \cdot AP$, find AP.

_____ **15** What is the value of $(1) + (1 + 2) + (1 + 2 + 3) + ... + (1 + 2 + ... + 49 + 50)$?