Team Number _____

MBMT Cantor Guts Round — Set 1

April 7, 2018

_____ **1** **[3]** Daniel is exactly one year younger than his friend David. If David was born in the year 2008, in what year was Daniel born?

_____ **2** **[3]** Mr. Pham flips three coins. What is the probability that no two coins show the same side?

_____ **3** **[3]** John has a sheet of white paper which is 3 cm in height and 4 cm in width. He wants to paint the sky blue and the ground green so the entire paper is painted. If the ground takes up a third of the page, how much space (in cm^2) does the sky take up?

_____ **4** **[3]** Jihang and Eric are busy fidget spinning. While Jihang spins his fidget spinner at 15 revolutions per second, Eric only manages 10 revolutions per second. How many total revolutions will the two have made after 5 continuous seconds of spinning?

_____ **5** **[3]** Find the last digit of

$$1333337777 \cdot 209347802 \cdot 3940704 \cdot 2309476091.$$

Team Number _____

MBMT Cantor Guts Round — Set 2

April 7, 2018

_____ **6** **[4]** Evan, Chloe, Rachel, and Joe are splitting a cake. Evan takes $\frac{1}{3}$ of the cake, Chloe takes $\frac{1}{4}$, Rachel takes $\frac{1}{5}$, and Joe takes $\frac{1}{6}$. There is $\frac{1}{x}$ of the original cake left. What is x?

_____ **7** **[4]** Pacman is a 330° sector of a circle of radius 4. Pacman has an eye of radius 1, located entirely inside Pacman. Find the area of Pacman, not including the eye.

_____ **8** **[4]** The sum of two prime numbers a and b is also a prime number. If $a < b$, find a.

_____ **9** **[4]** A bus has 54 seats for passengers. On the first stop, 36 people get onto an empty bus. Every subsequent stop, 1 person gets off and 3 people get on. After the last stop, the bus is full. How many stops are there?

_____ **10** **[4]** In a game, jumps are worth 1 point, punches are worth 2 points, and kicks are worth 3 points. The player must perform a sequence of 1 jump, 1 punch, and 1 kick. To compute the player's score, we multiply the 1st action's point value by 1, the 2nd action's point value by 2, the 3rd action's point value by 3, and then take the sum. For example, if we performed a punch, kick, jump, in that order, our score would be $1 \times 2 + 2 \times 3 + 3 \times 1 = 11$. What is the maximal score the player can get?

Team Number _____

MBMT Cantor Guts Round — Set 3

April 7, 2018

_____ **11** **[5]** 6 students are sitting around a circle, and each one randomly picks either the number 1 or 2. What is the probability that there will be two people sitting next to each other who pick the same number?

_____ **12** **[5]** You can buy a single piece of chocolate for 60 cents. You can also buy a packet with two pieces of chocolate for $1.00. Additionally, if you buy four single pieces of chocolate, the fifth one is free. What is the lowest amount of money you have to pay for 44 pieces of chocolate? Express your answer in dollars and cents (ex. $3.70).

_____ **13** **[5]** For how many integers k is there an integer solution x to the linear equation $kx + 2 = 14$?

_____ **14** **[5]** Ten teams face off in a swim meet. The boys teams and girls teams are ranked independently, each team receiving some number of positive integer points, and the final results are obtained by adding the points for the boys and the points for the girls. If Blair's boys got 7th place while the girls got 5th place (no ties), what is the best possible total rank for Blair?

_____ **15** **[5]** Arlene has a square of side length 1, an equilateral triangle with side length 1, and two circles with radius 1/6. She wants to pack her four shapes in a rectangle without items piling on top of each other. What is the minimum possible area of the rectangle?

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MBMT Cantor Guts Round — Set 4

April 7, 2018

_____ **16** **[7]** Let a, b, and c be real numbers. If $a^3 + b^3 + c^3 = 64$ and $a + b = 0$, what is the value of c?

_____ **17** **[7]** Bender always turns 60 degrees clockwise. He walks 3 meters, turns, walks 2 meters, turns, walks 1 meter, turns, walks 4 meters, turns, walks 1 meter, and turns. How many meters does Bender have to walk to get back to his original position?

_____ **18** **[7]** Guang has 4 identical packs of gummies, and each pack has a red, a blue, and a green gummy. He eats all the gummies so that he finishes one pack before going on to the next pack, but he never eats two gummies of the same color in a row. How many different ways can Guang eat the gummies?

_____ **19** **[7]** Find the sum of all digits q such that there exists a perfect square that ends in q.

_____ **20** **[7]** The numbers 5 and 7 are written on a whiteboard. Every minute Stev replaces the two numbers on the board with their sum and difference. After 2017 minutes the product of the numbers on the board is m. Find the number of factors of m.

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MBMT Cantor Guts Round — Set 5

April 7, 2018

_____ **21** **[9]** On the planet Alletas, $\frac{32}{33}$ of the people with silver hair have purple eyes and $\frac{8}{11}$ of the people with purple eyes have silver hair. On Alletas, what is the ratio of the number of people with purple eyes to the number of people with silver hair?

_____ **22** **[9]** Let P be a point on $y = -1$. Let the clockwise rotation of P by 60° about $(0, 0)$ be P'. Find the minimum possible distance between P' and $(0, -1)$.

_____ **23** **[9]** How many triangles can be made from the vertices and center of a regular hexagon? Two congruent triangles with different orientations are considered distinct.

_____ **24** **[9]** Jeremy and Kevin are arguing about how cool a sweater is on a scale of 1–5. Jeremy says "3", and Kevin says "4". Jeremy angrily responds "3.5", to which Kevin replies "3.75". The two keep going at it, responding with the average of the previous two ratings. What rating will they converge to (and settle on as the coolness of the sweater)?

_____ **25** **[9]** An even positive integer n has an *odd factorization* if the largest odd divisor of n is also the smallest odd divisor of n greater than 1. Compute the number of even integers n less than 50 with an *odd factorization*.

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MBMT Cantor Guts Round — Set 6

April 7, 2018

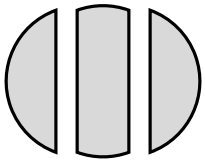
Note. Every answer in this section must be positive and given in decimal notation to receive points; 1000 and 4354.3 are allowed, while 2/3, -34, 0, and π are not.

Also, some of the problems are on the back of this sheet.

_____ **26** **[12]** When $2018! = 2018 \times 2017 \times \ldots \times 1$ is multiplied out and written as an integer, find the number of 4's.

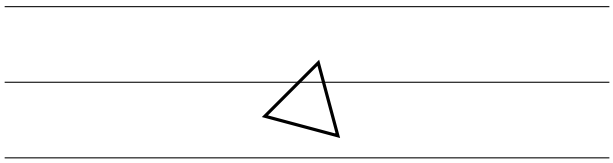
If the correct answer is A and your answer is E, you will receive $12 \min(A/E, E/A)^3$ points.

_____ **27** **[12]** A circle of radius 10 is cut into three pieces of equal area with two parallel cuts. Find the width of the center piece.



If the correct answer is A and your answer is E, you will receive $\max(0, 12 - 6|A - E|)$ points.

_____ **28** **[12]** An equilateral triangle of side length 1 is randomly thrown onto an infinite set of lines, spaced 1 apart.



On average, how many times will the boundary of the triangle intersect one of the lines? For example, in the above diagram, the boundary of the triangle intersects the lines in 2 places.

If the correct answer is A and your answer is E, you will receive $\max(0, 12-120|A-E|/A)$ points.

_____ **29** **[12]** Call an ordered triple of integers (a, b, c) *nice* if there exists an integer x such that $ax^2 + bx + c = 0$. How many *nice* triples are there such that $-100 \leq a, b, c \leq 100$?

If the correct answer is A and your answer is E, you will receive $12 \min(A/E, E/A)$ points.

_____ **30** **[12]** Let $f(i)$ denote the number of MBMT volunteers to be born in the ith state to join the United States. Find the value of $1f(1) + 2f(2) + 3f(3) + \cdots + 50f(50)$.
Note 1: Maryland was the 7th state to join the US.
Note 2: Last year's MBMT competition had 42 volunteers.

If the correct answer is A and your answer is E, you will receive $\max(0, 12 - 500(|A - E|/A)^2)$ points.