MBMT Geometry Round — Cantor

April 7, 2018

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

1 A circle has circumference 6π. Find the area of this circle.

Proposed by Daniel Zhu

Solution. $\boxed{9\pi}$

The circle has radius $\frac{6\pi}{2\pi} = 3$. Thus it has area $\pi \cdot 3^2 = 9\pi$. □

2 Points A, B, and C are on a line such that $AB = 6$ and $BC = 11$. Find all possible values of AC.

Proposed by Daniel Zhu

Solution. $\boxed{5, 17}$

Depending on the location of A, AC is either $11 + 6 = 17$ or $11 - 6 = 5$. □

3 A trapezoid has area 84 and one base of length 5. If the height is 12, what is the length of the other base?

Proposed by Steven Qu

Solution. $\boxed{9}$

If the length of the other base is b, then

$$\frac{1}{2} \cdot 12 \cdot (5 + b) = 84.$$

Solving this yields $b = \boxed{9}$ □

4 27 cubes of side length 1 are arranged to form a $3 \times 3 \times 3$ cube. If the corner $1 \times 1 \times 1$ cubes are removed, what fraction of the volume of the big cube is left?

Proposed by Chris Tong

Solution. $\boxed{\frac{19}{27}}$

There are 8 corners. If the 8 corner pieces are removed, then the resulting solid will contain $27 - 8 = 19$ 1 by 1 by 1 cubes. Therefore, the ratio of the volumes is $\boxed{\frac{19}{27}}$. □

5 There is a 50-foot tall wall and a 300-foot tall guard tower 50 feet from the wall. What is the minimum a such that a flat "X" drawn on the ground a feet from the side of the wall opposite the guard tower is visible from the top of the guard tower?

Proposed by Shwetha Kunnam

Solution. $\boxed{10}$

The tower and wall should line up at the maximum possible distance point; if the tower is further back, the "X" won't be visible, while if the tower is further forward, the distance will not be maximized. Set up similar triangles: one consists of the guard tower's top, base, and the "X" as vertices, while the other consists of the wall's top, base, and the "X" as vertices. Then, set up a proportion: (wall height)/(distance from wall to "X") = (tower height)/(distance from tower to "X") and substitute values to get 50/10 = 300/y. Solve to get y = 60, but remember that y = distance from tower to "X", so you subtract the distance from wall to "X" (10 feet) to get $\boxed{50}$ feet □

6 Steven's pizzeria makes pizzas in the shape of equilateral triangles. If a pizza with side length 8 inches will feed 2 people, how many people will a pizza of side length of 16 inches feed?

Proposed by Daniel Zhu

Solution. $\boxed{8}$

The area scales up by a factor $2^2 = 4$. So the answer isl $2 \cdot 4 = \boxed{8}$. □

7 Consider rectangle $ABCD$, with $1 = AB < BC$. The angle bisector of $\angle DAB$ intersects $\overline{BC}$ at E and $\overrightarrow{DC}$ at F. If $FE = FD$, find BC.

Proposed by Steven Qu

Solution. $\boxed{2+\sqrt{2}}$

Let $BC = x$.

$$FE = FA - EA = (x-1)\sqrt{2}, FD = AD = x.$$

Equating and solving yields $x = \boxed{2+\sqrt{2}}$. □

8 $\triangle ABC$ is a right triangle with $\angle A = 90^\circ$. Square $ADEF$ is drawn, with D on $\overline{AB}$, F on $\overline{AC}$, and E inside $\triangle ABC$. Point G is chosen on $\overline{BC}$ such that EG is perpendicular to BC. Additionally, $DE = EG$. Given that $\angle C = 20^\circ$, find the measure of $\angle BEG$.

Proposed by Kevin A. Zhou

Solution. $\boxed{55^\circ}$

Since the angles of a triangle add up to 180°, the $\angle B = 70^\circ$. Notice that point E is the incenter of triangle ABC. This means that BE bisects $\angle ABC$, so the measure of $\angle GBE = 35^\circ$. Then, the measure of $\angle BEG = \boxed{55^\circ}$. □