MBMT Counting and Probability Round — Cantor

April 7, 2018

Full Name ______________________________________

Team Number _________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

____________ **1** In Kevtown, license plates consist of a single uppercase letter followed by a single digit (like B6 or Y0). How many possible license plates can be made in Kevtown?

Proposed by Haydn Gwyn

Solution. $\boxed{260}$

There are 26 letters to choose from and 10 digits to choose from, for a total $26 \cdot 10 = \boxed{260}$ possible license plates. □

____________ **2** Artemis, Zeus, and Poseidon are betting on the outcome of rolling a six-sided fair die. Artemis bets that it will be a 2, Zeus bets that it will be odd, and Poseidon bets that it will be a multiple of 3. Who is most likely to be correct?

Proposed by Daniel Zhu

Solution. $\boxed{\text{Zeus}}$

Artemis is correct if a 2 is rolled, Zeus is correct if a 1, 3, or 5 is rolled, and Poseidon is correct if a 3 or 6 is rolled. Since all rolls are equally likely, then since Zeus is correct on the most rolls, $\boxed{\text{Zeus}}$ is most likely to be correct. □

____________ **3** Stan flips a penny, and Dilhan flips a nickel. If one of them flips tails, their coin vanishes. Otherwise, they keep the coin. What is the probability that they will have at least two cents left over?

Proposed by Kevin Qian

Solution. $\boxed{\dfrac{1}{2}}$

We can either have 0, 1, 5, or 6 cents after flipping. Notice that as long as Dilhan doesn't lose the nickel, we can afford the gum. Dilhan loses the nickel with chance $\frac{1}{3}$, so the answer is $1 - \frac{1}{2} = \boxed{\dfrac{1}{2}}$ □

____________ **4** A pie is covered with various toppings. Strawberries cover 50% of the pie, blueberries cover 40% of the pie, and 25% of the pie has neither topping. What percentage of the pie has both toppings?

Proposed by Kevin A. Zhou

Solution. $\boxed{15\%}$

Since 25% of the pie has neither topping, 75% of the pie has at least one topping. By the principle of inclusion and exclusion, the percentage of the pie with both toppings is $50\% + 40\% - 75\% = \boxed{15\%}$. □

_____ **5** David draws a square and puts a letter at each of its vertices. He looks at the square and reads $DAVE$. How many other names could he have possibly read (counting things like $EDAV$ or $ADEV$ which are not necessarily common names)?

Proposed by David Wu

Solution. $\boxed{7}$

There are 4 locations to start from and 2 possible directions to read the word in. If it is *not* "DAVE", then there are $4 \times 2 - 1 = \boxed{7}$ other possible words. □

_____ **6** Jimmy rolls a six-sided die over and over until he gets a number less than 5. Jommy rolls a six-sided die over and over until he gets a number greater than 2. Then, they each write down their number. What is the probability that the sum of their two numbers is equal to 7?

Proposed by Haydn Gwyn

Solution. $\boxed{\frac{1}{4}}$

There are four cases for Jimmy and Jommy's numbers that sum to 7: $(1, 6)$, $(2, 5)$, $(3, 4)$, and $(4, 3)$. Since Jimmy can get 1, 2, 3, or 4 (four cases), and Jommy can get 3, 4, 5, 6 (four cases), there are $4 \cdot 4 = 16$ total cases for Jimmy and Jommy's numbers, so the probability is $\frac{4}{16} = \frac{1}{4}$. □

_____ **7** How many ways are there to place one black piece and one white piece on a 3 by 3 checkerboard such that the pieces are in neither the same row nor the same column?

Proposed by Jyotsna Rao

Solution. $\boxed{36}$

There are 9 spaces to place the first castle, and then 4 spaces to place the second one. The answer is then $9 \cdot 4 = \boxed{36}$. □

_____ **8** Tom is stringing red, blue, and green beads on a straight wire. A red bead can be followed by any color of bead. A blue bead can only be followed by a blue or green bead. A green bead can only be followed by a green bead. If the wire has to have 6 beads, in how many ways can Tom string the beads? Note that not all colors have to be used.

Proposed by Jyotsna Rao

Solution. $\boxed{28}$

Notice that we will always have some number of reds, followed by some number of blues, followed by some number of greens, possibly 0 of some of the colors. Therefore, we can think of this as choosing the number of beads of each color. By stars and bars, there are $\binom{8}{2} = \boxed{28}$ ways. □