MBMT Algebra Round — Cantor

April 7, 2018

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

1 Guang has one pair of Yeezys. At the end of every six months, he buys three more pairs of Yeezys. How many months pass until Guang first has more than 10 pairs of Yeezys?

Proposed by David Wu

Solution. $\boxed{24}$

Let x be the number of six month periods that have passed. Then Guang has $1+3x$ number of Yeezys. If $1+3x > 10$ then $x \geq 4$. $x = 4$ corresponds to $4 \cdot 6 = \boxed{24}$ months. □

2 UMBC and UVA are playing a basketball game. If UMBC scores 12 three-pointers, 14 two-pointers, and 10 free throws (worth one point), while UVA scores 4 three-pointers, 19 two-pointers, and 4 free throws, then by how much did UMBC beat UVA?

Proposed by Kevin Qian

Solution. $\boxed{20}$

$$12 \times 3 + 14 \times 2 + 10 \times 1 - 4 \times 3 - 19 \times 2 - 4 \times 1 = \boxed{20}$$

□

3 To test Bob's memory, Alice tells Bob the numbers 1 through 10 in some order, but she skips one number. Bob is supposed to, in return, tell Alice the skipped number. Bob doesn't have a great memory, but he is clever, so he sums up the numbers Alice tells him. If Bob gets a sum of 50, what is the missing number?

Proposed by Guang Cui

Solution. $\boxed{5}$

If no number were missing, the total sum would be $1 + ... + 10 = 55$, so the missing number is $55 - 50 = \boxed{5}$. □

4 Julia and Jasmine evenly split a cake that is 30% frosting and 70% non-frosting. If Julia's portion is 50% frosting, what percent of Jasmine's portion is frosting?

Proposed by Steven Qu

Solution. $\boxed{10\%}$

If we let the cake be 100%, then Julia got 25% frosting and 25% non-frosting, leaving Jasmine with 5% frosting and 45% non-frosting, so overall Jasmine's portion is $\boxed{10\%}$ frosting. □

5 Guang is reading a book with 500 pages. Each day, he reads one page. On every Thursday, in addition to reading a new page, Guang re-reads (reviews) every page he's read so far. If each page takes one minute to read, how many minutes does Guang spend reading in ten weeks?

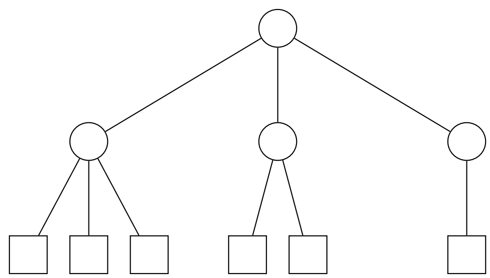
Proposed by Guang Cui

Solution. $\boxed{455}$

He will spend 70 minutes reading the first seven books. Also, he will spend $7 + 14 + \cdots + 70 = \boxed{455}$ minutes reviewing.

Note: The above solution erroneously assumed that the weeks began on a Friday. At the competition, we accepted any answer that resulted from starting on any day of the week. □

______ **6** Consider the following diagram. Steven places the numbers 1, 2, 3, 4, 5, and 6 in the squares, and writes in each circle the average of the numbers below it. What is the positive difference between the maximum and minimum possible values written in the top circle?



Proposed by Daniel Zhu

Solution. $\boxed{\frac{4}{3}}$

Clearly the top circle is just a weighted sum of the squares. So we can compute the maximum as $(\frac{6}{3} + \frac{9}{2} + 6)/3 = \frac{25}{6}$ and the minimum is $(\frac{15}{3} + \frac{5}{2} + 1)/3 = \frac{17}{6}$. The difference is $\frac{8}{6} = \boxed{\frac{4}{3}}$. □

______ **7** Guang's watch runs 1% slower than normal time. Luckily, he resets the time on his watch to be equal to the actual time at 6 AM, 11 AM, 4 PM, and 10 PM every day. What is the maximum difference in seconds ever achieved between the time on Guang's watch and the actual time?

Proposed by Daniel Zhu

Solution. $\boxed{288}$

Note that after a period of x hours, then Guang's watch will be $\frac{x}{100}$ hours off. Therefore the answer is $\frac{8}{100}$ hours $= \frac{480}{100}$ minutes $= \frac{24}{5}$ minutes $= 288$ seconds. □

______ **8** If $a^2 + 2b^2 = 72$ and $(a + 2b)^2 = 144$, and neither a nor b is equal to 0, find ab.

Proposed by Steven Qu

Solution. $\boxed{16}$

We have

$$(a+2b)^2 = a^2 + 4ab + 4b^2 == 2(72) = 2a^2 + 4b^2,$$

so

$$a^2 + 4ab + 4b^2 = 2a^2 + 4b^2 \implies 4ab = a^2$$

This implies either $4b = a$ or $a = 0$. Since $a \neq 0$, $a = 4b$. Now, we plug $4b = a$ back in to get $a = 8$ and $b = 2$, so the answer is $\boxed{16}$. □