MBMT Team Round — Ramanujan

April 1, 2017

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

DO NOT TURN THIS TEST IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

1 What is $11^2 - 9^2$?

Proposed by Pratik Rathore

Solution. $\boxed{40}$

$11^2 - 9^2 = (11 + 9)(11 - 9) = (20)(2) = \boxed{40}$.

Alternately, $11^2 = 121$ and $9^2 = 81$, so the answer is $121 - 81 = \boxed{40}$. □

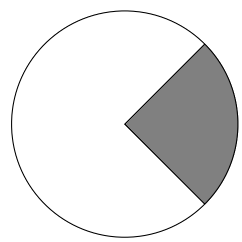
2 Write $\frac{9}{15}$ as a decimal.

Proposed by Guang Cui

Solution. $\boxed{0.6}$

$\frac{9}{15} = \frac{3}{5} = \boxed{0.6}$ □

3 A $90°$ sector of a circle is shaded, as shown below.



What percent of the circle is shaded?

Proposed by Guang Cui

Solution. $\boxed{25}$

One fourth if shaded, or $\boxed{25\%}$. □

4 A fair coin is flipped twice. What is the probability that the results of the two flips are different?

Proposed by Guang Cui

Solution. $\boxed{\dfrac{1}{2}}$

The results of the flips can be heads and then tails or tails and then heads. These are 2 possibilities over the $2^2 = 4$ equally probable outcomes. Therefore the answer is $\frac{2}{4} = \boxed{\frac{1}{2}}$. □

5 Wayne Dodson has 55 pounds of tungsten. If each ounce of tungsten is worth 75 cents, and there are 16 ounces in a pound, how much money, in dollars, is Wayne Dodson's tungsten worth?

Proposed by Eric Lu

Solution. $\boxed{\$660}$

$55 \cdot 0.75 \cdot 16 = 55 \cdot 12 = 110 \cdot 6 = \boxed{\$660}$. □

____________ **6** Tenley Towne has a collection of 28 sticks. With these 28 sticks he can build a tower that has 1 stick in the top row, 2 in the next row, and so on. Let n be the largest number of rows that Tenley Towne's tower can have. What is n?

Proposed by Sambuddha Chattopadhyay

Solution. $\boxed{7}$

We want to find the largest n such that $1 + 2 + ... + n \leq 28$. When $n = 7$, the left hand-side of the inequality is exactly 28. Thus the answer is $\boxed{7}$. □

____________ **7** What is the sum of the four smallest primes?

Proposed by Pratik Rathore

Solution. $\boxed{17}$

$2 + 3 + 5 + 7 = \boxed{17}$. □

____________ **8** Let ABC be an isosceles triangle such that $\angle B = 42^\circ$. What is the sum of all possible degree measures of angle A?

Proposed by Pratik Rathore

Solution. $\boxed{207}$

$m\angle A$ can either be 42, $\frac{180-42}{2} = 69$ or $180 - 42 \cdot 2 = 96$. The sum of these values is $\boxed{207}$. □

9 Consider a line passing through $(0,0)$ and $(4,8)$. This line passes through the point $(2,a)$. What is the value of a?

Proposed by Pratik Rathore

Solution. $\boxed{4}$

The slope of the line is $\frac{8-0}{4-0} = 2$. Using the slope, we find that the y-intercept is 0, so the equation of the line is $y = 2x$. So the line passes through $(2, 2 \cdot 2) = (2,4) \implies a = \boxed{4}$.

Alternately, 2 is the mean of 0 and 4, so the corresponding y-coordinate must be the mean of 0 and 8, which gives the same answer. □

10 Brian and Stan are playing a game. In this game, Brian rolls a fair six-sided die, while Stan rolls a fair four-sided die. Neither person shows the other what number they rolled. Brian tells Stan, "The number I rolled is guaranteed to be higher than the number you rolled." Stan now has to guess Brian's number. If Stan plays optimally, what is the probability that Stan correctly guesses the number that Brian rolled?

Proposed by Kevin Qian

Solution. $\boxed{\frac{1}{2}}$

The only way he's guaranteed to win is if the number is 5 or 6. Since each is equally likely, he has a $\boxed{\frac{1}{2}}$ chance of winning. □

11 Guang chooses 4 distinct integers between 0 and 9, inclusive. How many ways can he choose the integers such that every pair of chosen integers sums up to an even number?

Proposed by Dilhan Salgado

Solution. $\boxed{10}$

They can be all odd or all even, so there are $\binom{5}{4} + \binom{5}{4} = \boxed{10}$ ways to do this. □

12 David is trying to write a problem for MBMT. He assigns degree measures to every interior angle in a convex n-gon, and it so happens that every angle he assigned is less than 144 degrees. He tells Pratik the value of n and the degree measures in the n-gon, and to David's dismay, Pratik claims that such an n-gon does not exist. What is the smallest value of $n \geq 3$ such that Pratik's claim is necessarily true?

Proposed by David Wu

Solution. $\boxed{10}$

If every angle in a convex n-gon is less than $180 - 360/n$, then it is impossible for it to be an n-gon. For $n = 10$, we get $180 - 360/10 = 144$, and for $n = 9$ we get $180 - 360/9 = 140 < 144$, so $n = 9$ is possible. Thus the answer is $\boxed{10}$. □

13 Consider a triangle ABC with side lengths of 5, 5, and $2\sqrt{5}$. There exists a triangle with side lengths of 5, 5, and x ($x \neq 2\sqrt{5}$) which has the same area as ABC. What is the value of x?

Proposed by Pratik Rathore

Solution. $\boxed{4\sqrt{5}}$

Clearly triangle ABC is isosceles. The altitude to the side with length $2\sqrt{5}$ is $\sqrt{5^2 - \sqrt{5}^2} = 2\sqrt{5}$. If we rotate the two triangles formed by the altitude and put the $\sqrt{5}$ sides together, we get a triangle with sidelengths of 5, 5, and $4\sqrt{5}$. Therefore $x = \boxed{4\sqrt{5}}$. □

14 A mother has 11 identical apples and 9 identical bananas to distribute among her 3 kids. In how many ways can the fruits be allocated so that each child gets at least one apple and one banana?

Proposed by Jyotsna Rao

Solution. $\boxed{1260}$

The mother can first distribute one apple and one banana to each child to ensure that each child gets at least one of each type of fruit. This leaves 8 apples and 6 bananas. Using stars and bars, there are $\binom{10}{2}$ ways to distribute the remaining apples and $\binom{8}{2}$ ways to distribute the remaining bananas. The answer is thus $\binom{10}{2}\binom{8}{2} = \boxed{1260}$. □

15 Find the sum of the five smallest positive integers that cannot be represented as the sum of two not necessarily distinct primes.

Proposed by Dilhan Salgado

Solution. $\boxed{34}$

1,2, and 3 trivially cannot be represented as the sum of two primes. The rest of the even integers up to some very large number have been verified to work. For an odd number to work it must be 2+a prime, so the ones that fail are 2+odd composite. The first two of these are $2+9 = 11$ and $2+15 = 17$. The answer is thus $1+2+3+11+17 = \boxed{34}$. □