MBMT Team Round — Ramanujan

April 1, 2017

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.
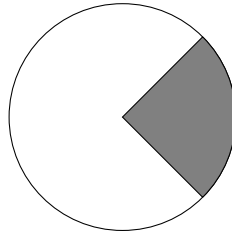
DO NOT TURN THIS TEST IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

_____ **1** What is $11^2 - 9^2$?

_____ **2** Write $\frac{9}{15}$ as a decimal.

_____ **3** A 90° sector of a circle is shaded, as shown below.

What percent of the circle is shaded?

_____ **4** A fair coin is flipped twice. What is the probability that the results of the two flips are different?

_____ **5** Wayne Dodson has 55 pounds of tungsten. If each ounce of tungsten is worth 75 cents, and there are 16 ounces in a pound, how much money, in dollars, is Wayne Dodson's tungsten worth?

_____ **6** Tenley Towne has a collection of 28 sticks. With these 28 sticks he can build a tower that has 1 stick in the top row, 2 in the next row, and so on. Let n be the largest number of rows that Tenley Towne's tower can have. What is n?

_____ **7** What is the sum of the four smallest primes?

_____ **8** Let ABC be an isosceles triangle such that $\angle B = 42°$. What is the sum of all possible degree measures of angle A?

9 Consider a line passing through $(0,0)$ and $(4,8)$. This line passes through the point $(2, a)$. What is the value of a?

10 Brian and Stan are playing a game. In this game, Brian rolls a fair six-sided die, while Stan rolls a fair four-sided die. Neither person shows the other what number they rolled. Brian tells Stan, "The number I rolled is guaranteed to be higher than the number you rolled." Stan now has to guess Brian's number. If Stan plays optimally, what is the probability that Stan correctly guesses the number that Brian rolled?

11 Guang chooses 4 distinct integers between 0 and 9, inclusive. How many ways can he choose the integers such that every pair of chosen integers sums up to an even number?

12 David is trying to write a problem for MBMT. He assigns degree measures to every interior angle in a convex n-gon, and it so happens that every angle he assigned is less than 144 degrees. He tells Pratik the value of n and the degree measures in the n-gon, and to David's dismay, Pratik claims that such an n-gon does not exist. What is the smallest value of $n \geq 3$ such that Pratik's claim is necessarily true?

13 Consider a triangle ABC with side lengths of 5, 5, and $2\sqrt{5}$. There exists a triangle with side lengths of 5, 5, and x ($x \neq 2\sqrt{5}$) which has the same area as ABC. What is the value of x?

14 A mother has 11 identical apples and 9 identical bananas to distribute among her 3 kids. In how many ways can the fruits be allocated so that each child gets at least one apple and one banana?

15 Find the sum of the five smallest positive integers that cannot be represented as the sum of two not necessarily distinct primes.