MBMT Number Theory Round — Ramanujan

April 1, 2017

Full Name ____

Team Number _________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered by fewer competitors will be weighted more heavily. Please write your answers in the simplest possible form.

____________ **1** What is the smallest integer greater than 10 that leaves a remainder of 1 when divided by 4?

Proposed by Eric Lu

Solution. $\boxed{13}$

The numbers that leave a remainder of 1 when divided by 4 are one more than multiples of 4, so 1, 5, 9, 13, 17,... are the numbers that satisfy the condition. The smallest number larger than ten in this list is $\boxed{13}$. □

____________ **2** The sequence 5, 7, 11, 19, 35, . . . is formed by multiplying the previous term by 2 and subtracting 3. What is the 6th term in the sequence?

Proposed by Guang Cui

Solution. $\boxed{67}$

The next term in this sequence is formed by multiplying 3 (the last given term) by 2 and subtracting 3. This gives us $2 \cdot 35 - 3 = 70 - 3 = \boxed{67}$. □

____________ **3** How many integers between 1 and 100 inclusive are divisible by 4?

Proposed by Pratik Rathore

Solution. $\boxed{25}$

Since the smallest multiple of 4 in the range is $4 \cdot 1$ and the largest multiple of 4 in the range is $4 \cdot 25$, the answer is $25 - 1 + 1 = \boxed{25}$. □

____________ **4** What is the greatest common factor of 91 and 78?

Proposed by David Wu

Solution. $\boxed{13}$

Without using the Euclidean Algorithm, we just factor 91 and 78. $91 = 7 \cdot 13$, and $78 = 6 \cdot 13$. Hence the answer is $\boxed{13}$. □

____________ **5** Let $\overline{201A}$ be a four-digit number that is divisible by 3. Find the sum of all possible values of A.

Proposed by Pratik Rathore

Solution. $\boxed{18}$

Since the sum of the digits of the number is $3 + A$, A can be any multiple of 3, so the answer is $0 + 3 + 6 + 9 = \boxed{18}$. □

____________ **6** The sum of two prime numbers is 30. Find the largest possible product of the two primes.

Proposed by Guang Cui

Solution. $\boxed{221}$

To maximize the product, the two primes should be as close to each other as possible. It turns out that 13 and 17 are the two primes we want, since they are the closest to $\frac{30}{2} = 15$. Thus the answer is $13 \cdot 17 = \boxed{221}$. □

__________ **7** Guang loves having Mighty Wings and Shamrock Shakes at McDonalds. He orders Mighty Wings every 3 days and Shamrock Shakes every 4 days. In a period of 28 consecutive days, what is the most number of days where he orders both Mighty Wings and Shamrock Shakes?

Proposed by David Wu

Solution. $\boxed{3}$

Clearly, there are exactly 12 days that pass between days where he orders both. It's easy to see that the most optimal arrangement would be for Guang to order both on the first of the 28 days, then the thirteenth, then the twenty-fifth. Thus the answer is $\boxed{3}$. □

__________ **8** Mr. Stein wants to buy some Munchkins from Dunkin' Donuts. They sell Munchkins in packages of 5 and 8. What is the largest integer number of Munchkins that Mr. Stein can't buy?

Proposed by Annie Zhao

Solution. $\boxed{27}$

Since 5 and 8 are relatively prime, we can use the Chicken McNugget Theorem. Therefore, the answer is $40 - 5 - 8 = \boxed{27}$.

Alternately, one can test possibilities for each number of munchkins until 5 consecutive values work. If 5 straight values work, then we can always add another package of 5 Munchkins to get the next 5 numbers (e.g. $\{30, 31, 32, 33, 34\} \Longrightarrow \{35, 36, 37, 38, 39\}$). By testing, one can see that there are not 5 consecutive numbers less than 27 that all work, and neither does 27 itself. However, the five numbers after 27 do work:

$$28 = 5 \cdot 4 + 8 \cdot 1$$
$$29 = 5 \cdot 1 + 8 \cdot 3$$
$$30 = 5 \cdot 6 + 8 \cdot 0$$
$$31 = 5 \cdot 3 + 8 \cdot 2$$
$$32 = 5 \cdot 0 + 8 \cdot 4$$

Thus $\boxed{27}$ is the answer. □