

MBMT Ramanujan Guts Round – Set 1

April 1, 2017

_____ 1 [3] Find $291 + 503 - 91 + 492 - 103 - 392$.

_____ 2 [3] Let the operation $a \& b$ be defined to be $\frac{a-b}{a+b}$. What is $3 \& -2$?

_____ 3 [3] Joe can trade 5 apples for 3 oranges, and trade 6 oranges for 5 bananas. If he has 20 apples, what is the largest number of bananas he can trade for?

_____ 4 [3] A cone has a base with radius 3 and a height of 5. What is its volume? Express your answer in terms of π .

_____ 5 [3] Guang brought dumplings to school for lunch, but by the time his lunch period comes around, he only has two dumplings left! He tries to remember what happened to the dumplings. He first traded $\frac{3}{4}$ of his dumplings for Arman's samosas, then he gave 3 dumplings to Anish, and lastly he gave David $\frac{1}{2}$ of the dumplings he had left. How many dumplings did Guang bring to school?

MBMT Ramanujan Guts Round – Set 2

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_____ 6 [4] In the recording studio, Kanye has 10 different beats, 9 different manuscripts, and 8 different samples. If he must choose 1 beat, 1 manuscript, and 1 sample for his new song, how many selections can he make?

_____ 7 [4] How many lines of symmetry does a regular dodecagon (a polygon with 12 sides) have?

_____ 8 [4] Let there be numbers a, b, c such that $ab = 3$ and $abc = 9$. What is the value of c ?

_____ 9 [4] How many odd composite numbers are there between 1 and 20?

_____ 10 [4] Consider the line given by the equation $3x - 5y = 2$. David is looking at another line of the form $ax - 15y = 5$, where a is a real number. What is the value of a such that the two lines do not intersect at any point?

MBMT Ramanujan Guts Round – Set 3

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_____ 11 [5] Let $ABCD$ be a rectangle such that $AB = 4$ and $BC = 3$. What is the length of BD ?

_____ 12 [5] Daniel is walking at a constant rate on a 100-meter long moving walkway. The walkway moves at 3 m/s. If it takes Daniel 20 seconds to traverse the walkway, find his walking speed (excluding the speed of the walkway) in m/s.

_____ 13 [5] Pratik has a 6 sided die with the numbers 1, 2, 3, 4, 6, and 12 on the faces. He rolls the die twice and records the two numbers that turn up on top. What is the probability that the product of the two numbers is less than or equal to 12?

_____ 14 [5] Find the two-digit number such that the sum of its digits is twice the product of its digits.

_____ 15 [5] If $a^2 + 2a = 120$, what is the value of $2a^2 + 4a + 1$?

MBMT Ramanujan Guts Round – Set 4

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- _____ 16 [7] Adam and Becky are building a house. Becky works twice as fast as Adam does, and they both work at constant speeds for the same amount of time each day. They plan to finish building in 6 days. However, after 2 days, their friend Charlie also helps with building the house. Because of this, they finish building in just 5 days. What fraction of the house did Adam build?
- _____ 17 [7] A bag with 10 items contains both pencils and pens. Kanye randomly chooses two items from the bag, with replacement. Suppose the probability that he chooses 1 pen and 1 pencil is $\frac{21}{50}$. What are all possible values for the number of pens in the bag?
- _____ 18 [7] In cyclic quadrilateral $ABCD$, $\angle ABD = 40^\circ$, and $\angle DAC = 40^\circ$. Compute the measure of $\angle ADC$ in degrees. (In cyclic quadrilaterals, opposite angles sum up to 180° .)
- _____ 19 [7] There is a strange random number generator which always returns a positive integer between 1 and 7500, inclusive. Half of the time, it returns a uniformly random positive integer multiple of 25, and the other half of the time, it returns a uniformly random positive integer that isn't a multiple of 25. What is the probability that a number returned from the generator is a multiple of 30?
- _____ 20 [7] Julia is shopping for clothes. She finds T different tops and S different skirts that she likes, where $T \geq S > 0$. Julia can either get one top and one skirt, just one top, or just one skirt. If there are 50 ways in which she can make her choice, what is $T - S$?

MBMT Ramanujan Guts Round – Set 5

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- _____ 21 [9] A $5 \times 5 \times 5$ cube's surface is completely painted blue. The cube is then completely split into $1 \times 1 \times 1$ cubes. What is the average number of blue faces on each $1 \times 1 \times 1$ cube?
- _____ 22 [9] Find the number of values of n such that a regular n -gon has interior angles with integer degree measures.
- _____ 23 [9] 4 positive integers form an geometric sequence. The sum of the 4 numbers is 255, and the average of the second and the fourth number is 102. What is the smallest number in the sequence?
- _____ 24 [9] Let S be the set of all positive integers which have three digits when written in base 2016 and two digits when written in base 2017. Find the size of S .
- _____ 25 [9] In square $ABCD$ with side length 13, point E lies on segment CD . Segment AE divides $ABCD$ into triangle ADE and quadrilateral $ABCE$. If the ratio of the area of ADE to the area of $ABCE$ is $4 : 11$, what is the ratio of the perimeter of ADE to the perimeter of $ABCE$?

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_____ 26 [12] Submit a decimal n to the nearest thousandth between 0 and 200. Your score will be $\min(12, S)$, where S is the non-negative difference between n and the largest number less than or equal to n chosen by another team (if you choose the smallest number, $S = n$). For example, 1.414 is an acceptable answer, while $\sqrt{2}$ and 1.4142 are not.

_____ 27 [12] Guang is going hard on his YNA project. From 1:00 AM Saturday to 1:00 AM Sunday, the probability that he is not finished with his project x hours after 1:00 AM on Saturday is $\frac{1}{x+1}$. If Guang does not finish by 1:00 AM on Sunday, he will stop procrastinating and finish the project immediately. Find the expected number of minutes A it will take for him to finish his project.

An estimate of E will earn $12 \cdot 2^{-|E-A|/60}$ points.

_____ 28 [12] All the diagonals of a regular 100-gon (a regular polygon with 100 sides) are drawn. Let A be the number of distinct intersection points between all the diagonals. Find A .

An estimate of E will earn $12 \cdot (16 \log_{10}(\max(\frac{E}{A}, \frac{A}{E})) + 1)^{-\frac{1}{2}}$ or 0 points if this expression is undefined.

_____ 29 [12] Find the smallest positive integer A such that the following is true: if every integer $1, 2, \dots, A$ is colored either red or blue, then no matter how they are colored, there are always 6 integers among them forming an increasing arithmetic progression that are all colored the same color.

An estimate of E will earn $12 \min(\frac{E}{A}, \frac{A}{E})$ points or 0 points if this expression is undefined.

_____ 30 [12] For all integers $n \geq 2$, let $f(n)$ denote the smallest prime factor of n . Find

$$A = \sum_{n=2}^{10^6} f(n).$$

In other words, take the smallest prime factor of every integer from 2 to 10^6 and sum them all up to get A .

You may find the following values helpful: there are 78498 primes below 10^6 , 9592 primes below 10^5 , 1229 primes below 10^4 , and 168 primes below 10^3 .

An estimate of E will earn $\max(0, 12 - 4 \log_{10}(\max(\frac{E}{A}, \frac{A}{E})))$ or 0 points if this expression is undefined.