MBMT Geometry Round — Ramanujan

April 1, 2017

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered by fewer competitors will be weighted more heavily. Please write your answers in the simplest possible form.

1 What is the distance between the points $(6,0)$ and $(-2,0)$?

Proposed by Mr. Stein

Solution. $\boxed{8}$

They are on a horizontal line so the distance is just the positive difference in x-coordinates, or $6-(-2)=\boxed{8}$. □

2 Angle X has a degree measure of 35 degrees. What is the supplement of the complement of angle X?

The complement of an angle is 90 degrees minus the angle measure. The supplement of an angle is 180 degrees minus the angle measure.

Proposed by David Wu

Solution. $\boxed{125^\circ}$

The complement of X is $90-35=55^\circ$. The supplement of 55° is $180-55=\boxed{125^\circ}$. □

3 A cube has a volume of 729. What is the side length of the cube?

Proposed by Mr. Stein

Solution. $\boxed{9}$

Let the side length of the cube be s. Then $s^3=729 \implies s=\boxed{9}$. □

4 A car that always travels in a straight line starts at the origin and goes towards the point $(8,12)$. The car stops halfway on its path, turns around, and returns back towards the origin. The car again stops halfway on its return. What are the car's final coordinates?

Proposed by David Wu

Solution. $\boxed{(2,3)}$

We use the midpoint formula twice to solve the problem. On the first path, the car stops at $(\frac{0+8}{2},\frac{0+12}{2})=(4,6)$. On the path back towards the origin, the car stops at $(\frac{4+0}{2},\frac{6+0}{2})=\boxed{(2,3)}$, which is the final answer. □

5 A full, cylindrical soup can has a height of 16 and a circular base of radius 3. All the soup in the can is used to fill a hemispherical bowl to its brim. What is the radius of the bowl?

Proposed by Jyotsna Rao

Solution. $\boxed{6}$

Let the radius of the can be r, the height of the can h, and the radius of the bowl x. The volume of the can is $\pi r^2 h=\pi\cdot 9\cdot 16=144\pi$. The volume of the bowl is $(\frac{4}{3}\pi x^3)/2=\frac{2}{3}\pi x^3$. This is equal to the volume of the can, so $\frac{2}{3}\pi x^3=144\pi$, implying $x^3=216$, whence $x=\boxed{6}$. □

6 In square $ABCD$, the numerical value of the length of the diagonal is three times the numerical value of the area of the square. What is the side length of the square?

Proposed by David Wu

Solution. $\boxed{\dfrac{\sqrt{2}}{3}}$

Let the side length of the square be s. Then $s \cdot \sqrt{2} = 3s^2 \implies s = \boxed{\dfrac{\sqrt{2}}{3}}$. □

7 Consider triangle ABC with $AB = 3$, $BC = 4$, and $AC = 5$. The altitude from B to AC intersects AC at H. Compute BH.

Proposed by Pratik Rathore

Solution. $\boxed{\dfrac{12}{5}}$

Since $3^2 + 4^2 = 5^2$, ABC is a right triangle. Thus $[ABC]$ is $\frac{3 \cdot 4}{2} = 6$. But since BH is an altitude, we know that $\frac{5 \cdot BH}{2} = [ABC] = 6 \implies BH = \boxed{\dfrac{12}{5}}$. □

8 Mary shoots 5 darts at a square with side length 2. Let x be equal to the shortest distance between any pair of her darts. What is the maximum possible value of x?

Proposed by Cynthia Liu

Solution. $\boxed{\sqrt{2}}$

Break up the square into 4 unit squares. By pigeonhole, at least 2 of the 5 darts will be in 1 of those unit squares. Therefore the longest shortest distance is the longest distance between two points in a unit square: the diagonal. The length is thus $\boxed{\sqrt{2}}$. □