MBMT Counting and Probability Round — Ramanujan

April 1, 2017

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered by fewer competitors will be weighted more heavily. Please write your answers in the simplest possible form.

______ **1** A machine takes in two numbers and gives 3 times their sum. Pratik gives the machine the numbers 101 and 99. What number does the machine give to Pratik?

Proposed by Pratik Rathore

Solution. $\boxed{600}$

$3(101 + 99) = 3 \cdot 200 = \boxed{600}$ □

______ **2** There are 5 red balls, 2 blue balls, and 3 green balls in a bag. If someone randomly takes out a ball from the bag without looking, what is the probability that it is not green?

Proposed by Guang Cui

Solution. $\boxed{\dfrac{7}{10}}$

There are 7 balls that are not green and 10 balls in total, resulting in an answer of $\boxed{\frac{7}{10}}$. □

______ **3** 4 people are at a party, and every pair of people shakes hands with each other. How many total handshakes are there?

Proposed by Mr. Stein

Solution. $\boxed{6}$

It suffices to count the number of pairs of people. To count this, we can choose one person to be the first person in the pair, which can be done in 4 ways, and a second person who is not the first, which can be done in 3 ways. However, this way each pair is counted twice, since pairs are not ordered. Therefore, the answer is $\frac{4 \cdot 3}{2} = \boxed{6}$.

Alternatively note that the answer is $\binom{4}{2} = \boxed{6}$. □

______ **4** What is the minimum number of balls one must take out of a bag containing 11 red balls, 7 yellow balls, and 6 blue balls to guarantee that at least one ball of each color has been taken out?

Proposed by Guang Cui

Solution. $\boxed{19}$

If one color of ball is never drawn, there must be at least 6 balls that are not drawn, since each color has at least 6 balls of that color. Therefore, since there are 24 balls in total, the maximum number of balls that do not contain every color is $24 - 6 = 18$. Thus $\boxed{19}$ balls are needed. □

______ **5** Timmy Turner has 4 burners, but one of them is broken. If he chooses two of his burners, what is the probability that both of them are not broken?

Proposed by Guang Cui

Solution. $\boxed{\frac{1}{2}}$

There are $\binom{3}{2} = 3$ ways to choose non-broken ones, and $\binom{4}{2} = 6$ total, so the answer is $\frac{3}{6} = \boxed{\frac{1}{2}}$. □

6 Dr. Dresnoopdogg is listening to his iPodZune through his BeatsByBose headset. He has 6 songs that he can play, and he can play them in any order. In how many possible orders can Dr. Dresnoopdogg play each of his songs once?

Proposed by Sambuddha Chattopadhyay

Solution. $\boxed{720}$

There are $6! = \boxed{720}$ ways to order his songs. □

7 A palindrome is a nonnegative integer that reads the same forwards and backwards. For example, 12321 and 0 are both palindromes. Find the number of palindromes between 1000 and 9999, inclusive.

Proposed by David Wu

Solution. $\boxed{90}$

Four-digit palindromes are of the form $\overline{abba}$, so we just need the number of ordered pairs of digits (a, b). Since $a \neq 0$, there are $9 \cdot 10$ such pairs, and the answer is $\boxed{90}$. □

8 Stan has five friends: Allen, Brian, Catherine, Daniel, and Evan. Each of the 6 people took a test and the teacher told each of them their own score privately. Now they want to share their scores with each other. In a conversation, Person A tells Person B all the scores they know, but not vice versa. What is the minimum number of conversations required for every person to know all six scores?

Proposed by Kevin Qian

Solution. $\boxed{10}$

Consider the earliest moment when someone knows everyone else's score. WLOG let this person be Stan. There must have been 5 conversations beforehand because each of the other five people must have revealed their score to someone else. On the other hand, this first person is unique, and we require at least 5 turns to inform each of the other people. Hence, the minimum is $\boxed{10}$. We can achieve 10 by having everyone tell Stan their score and have Stan distribute the information. □