MBMT Team Round — Pascal

April 1, 2017

Full Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

DO NOT TURN THIS TEST IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

____________ **1** Let ABC be an isosceles triangle such that $\angle B = 42^\circ$. What is the sum of all possible degree measures of angle A?

____________ **2** Brian and Stan are playing a game. In this game, Brian rolls a fair six-sided die, while Stan rolls a fair four-sided die. Neither person shows the other what number they rolled. Brian tells Stan, "The number I rolled is guaranteed to be higher than the number you rolled." Stan now has to guess Brian's number. If Stan plays optimally, what is the probability that Stan correctly guesses the number that Brian rolled?

____________ **3** Consider a triangle ABC with side lengths of 5, 5, and $2\sqrt{5}$. There exists a triangle with side lengths of 5, 5, and x ($x \neq 2\sqrt{5}$) which has the same area as ABC. What is the value of x?

____________ **4** David is trying to write a problem for MBMT. He assigns degree measures to every interior angle in a convex n-gon, and it so happens that every angle he assigned is less than 144 degrees. He tells Pratik the value of n and the degree measures in the n-gon, and to David's dismay, Pratik claims that such an n-gon does not exist. What is the smallest value of $n \geq 3$ such that Pratik's claim is necessarily true?

____________ **5** A mother has 11 identical apples and 9 identical bananas to distribute among her 3 kids. In how many ways can the fruits be allocated so that each child gets at least one apple and one banana?

____________ **6** Srinivasa Ramanujan has the polynomial $P(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$. His friend Hardy tells him that 3 is one of the roots of $P(x)$. What is the sum of the other roots of $P(x)$?

____________ **7** Find the sum of the five smallest positive integers that cannot be represented as the sum of two not necessarily distinct primes.

____________ **8** ABC is an equilateral triangle with side length 10. Let P be a point which lies on ray $\overrightarrow{BC}$ such that $PB = 20$. Compute the ratio $\frac{PA}{PC}$.

____________ **9** Let ABC be a triangle such that $AB = 10$, $BC = 14$, and $AC = 6$. The median CD and angle bisector CE are both drawn to side AB. What is the ratio of the area of triangle CDE to the area of triangle ABC?

____________ **10** Find all integer values of x between 0 and 2017 inclusive, which satisfy

$$2016x^{2017} + 990x^{2016} + 2x + 17 \equiv 0 \pmod{2017}.$$

____________ **11** Let $x^2 + ax + b$ be a quadratic polynomial with positive integer roots such that $a^2 - 2b = 97$. Compute $a + b$.

____________ **12** Let S be the set $\{2, 3, \ldots, 14\}$. We assign a distinct number from S to each side of a six-sided die. We say a numbering is predictable if prime numbers are always opposite prime numbers and composite numbers are always opposite composite numbers. How many predictable numberings are there? (Rotations of a die are not distinct)

____________ **13** In triangle ABC, $AB = 10$, $BC = 21$, and $AC = 17$. D is the foot of the altitude from A to BC, E is the foot of the altitude from D to AB, and F is the foot of the altitude from D to AC. Find the area of the smallest circle that contains the quadrilateral $AEDF$.

____________ **14** What is the greatest distance between any two points on the graph of

$$3x^2 + 4y^2 + z^2 - 12x + 8y + 6z = -11?$$

____________ **15** For a positive integer n, $\tau(n)$ is defined to be the number of positive divisors of n. Given this information, find the largest positive integer n less than 1000 such that

$$\sum_{d|n} \tau(d) = 108$$

In other words, we take the sum of $\tau(d)$ for every positive divisor d of n, which has to be 108.