# MBMT Number Theory Round — Pascal

**April 1, 2017**

Full Name _____

Team Number _____

# DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered by fewer competitors will be weighted more heavily. Please write your answers in the simplest possible form.

**1** What is the greatest common factor of 91 and 78?

**2** Let $\overline{201A}$ be a four-digit number that is divisible by 3. Find the sum of all possible values of $A$.

**3** How many three-digit positive integers are divisible either by 3 or by 7, but not by both?

**4** Benedict Arnold is a confused man. If he eats a George Washington cake he loses a traitor point. But if he eats a George Wilhelm cake he doubles his traitor points. If he reaches exactly 2017 traitor points, Ethan Allen won't buy him furniture. If Benedict Arnold starts out with 1 traitor point, what is the minimum number of cakes he must eat so that Ethan Allen won't buy him furniture?

**5** For all positive integers $n$, let the multiplicative average of $n$ be the geometric mean of all the positive divisors of $n$. (The geometric mean of positive reals $x_1, x_2, \ldots, x_k$ is $\sqrt[k]{x_1 \cdot x_2 \cdots x_k}$). Let $S$ be the number of positive integers $n$ such that the multiplicative average of $n$ is less than or equal to 2017. Find the remainder when $S$ is divided by 1000.

**6** Let $S$ be the set of all positive integers less than or equal to 100. Guang randomly chooses two not-necessarily distinct elements of $S$ and finds their greatest common divisor, $d$. What is the probability that $d$ has exactly 12 factors?

**7** Let the set $S$ contain all ordered triples of positive integers $(x, y, z)$ satisfying

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2018}{2017}$$

Compute the remainder when

$$\sum_{(x,y,z) \in S} x + y + z$$

is divided by 1000. In other words, find the sum of all $x + y + z$ over the ordered triples in $S$, and find the remainder when this value is divided by 1000.

**8** Shane has an infinite set of cards labeled $1, 1, 26, 26, 676, 676, \ldots, 26^k, 26^k, \ldots$ for all non-negative integers $k$. In other words, he has two cards for each integer power of 26. Shane chooses a non-empty set of cards and finds the sum of the numbers on the cards that he has selected. Let the set $S$ contain all of the distinct sums that Shane can make. What is the remainder when the 2017th smallest element in $S$ divided by 1000?