Team Number _____

MBMT Pascal Guts Round — Set 1

April 1, 2017

__________ **1** **[3]** Find $291 + 503 - 91 + 492 - 103 - 392$.

__________ **2** **[3]** In the recording studio, Kanye has 10 different beats, 9 different manuscripts, and 8 different samples. If he must choose 1 beat, 1 manuscript, and 1 sample for his new song, how many selections can he make?

__________ **3** **[3]** Pratik has a 6 sided die with the numbers 1, 2, 3, 4, 6, and 12 on the faces. He rolls the die twice and records the two numbers that turn up on top. What is the probability that the product of the two numbers is less than or equal to 12?

__________ **4** **[3]** Adam and Becky are building a house. Becky works twice as fast as Adam does, and they both work at constant speeds for the same amount of time each day. They plan to finish building in 6 days. However, after 2 days, their friend Charlie also helps with building the house. Because of this, they finish building in just 5 days. What fraction of the house did Adam build?

__________ **5** **[3]** Find the two-digit number such that the sum of its digits is twice the product of its digits.

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MBMT Pascal Guts Round — Set 2

April 1, 2017

__________ **6** **[4]** There is a strange random number generator which always returns a positive integer between 1 and 7500, inclusive. Half of the time, it returns a uniformly random positive integer multiple of 25, and the other half of the time, it returns a uniformly random positive integer that isn't a multiple of 25. What is the probability that a number returned from the generator is a multiple of 30?

__________ **7** **[4]** Julia is shopping for clothes. She finds T different tops and S different skirts that she likes, where $T \geq S > 0$. Julia can either get one top and one skirt, just one top, or just one skirt. If there are 50 ways in which she can make her choice, what is $T - S$?

__________ **8** **[4]** In cyclic quadrilateral $ABCD$, $\angle ABD = 40^\circ$, and $\angle DAC = 40^\circ$. Compute the measure of $\angle ADC$ in degrees. (In cyclic quadrilaterals, opposite angles sum up to 180°.)

__________ **9** **[4]** 4 positive integers form an geometric sequence. The sum of the 4 numbers is 255, and the average of the second and the fourth number is 102. What is the smallest number in the sequence?

__________ **10** **[4]** Let S be the set of all positive integers which have three digits when written in base 2016 and two digits when written in base 2017. Find the size of S.

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MBMT Pascal Guts Round — Set 3

April 1, 2017

___________ **11** **[5]** Find all possible values of c in the following system of equations:

$$a^2 + ab + c^2 = 31$$
$$b^2 + ab - c^2 = 18$$
$$a^2 - b^2 = 7.$$

___________ **12** **[5]** In square $ABCD$ with side length 13, point E lies on segment CD. Segment AE divides $ABCD$ into triangle ADE and quadrilateral $ABCE$. If the ratio of the area of ADE to the area of $ABCE$ is $4 : 11$, what is the ratio of the perimeter of ADE to the perimeter of $ABCE$?

___________ **13** **[5]** Thomas has two distinct chocolate bars. One of them is 1 by 5 and the other one is 1 by 3. If he can only eat a single 1 by 1 piece off of either the leftmost side or the rightmost side of either bar at a time, how many different ways can he eat the two bars?

___________ **14** **[5]** In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. The entire triangle is revolved about side BC. What is the volume of the swept out region?

___________ **15** **[5]** Find the number of ordered pairs of positive integers (a, b) that satisfy the equation $a(a - 1) + 2ab + b(b - 1) = 600$.

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MBMT Pascal Guts Round — Set 4

April 1, 2017

__________ **16** **[7]** Compute the sum of the digits of $(10^{2017} - 1)^2$.

__________ **17** **[7]** A right triangle with area 210 is inscribed within a semicircle, with its hypotenuse coinciding with the diameter of the semicircle. 2 semicircles are constructed (facing outwards) with the legs of the triangle as their diameters. What is the area inside the 2 semicircles but outside the first semicircle?

__________ **18** **[7]** Find the smallest positive integer n such that exactly $\frac{1}{10}$ of its positive divisors are perfect squares.

__________ **19** **[7]** One day, Sambuddha and Jamie decide to have a tower building competition using oranges of radius 1 inch. Each player begins with 14 oranges. Jamie builds his tower by making a 3 by 3 base, placing a 2 by 2 square on top, and placing the last orange at the very top. However, Sambuddha is very hungry and eats 4 of his oranges. With his remaining 10 oranges, he builds a similar tower, forming an equilateral triangle with 3 oranges on each side, placing another equilateral triangle with 2 oranges on each side on top, and placing the last orange at the very top. What is the positive difference between the heights of these two towers?

__________ **20** **[7]** Let r, s, and t be the roots of the polynomial $x^3 - 9x + 42$. Compute the value of $(rs)^3 + (st)^3 + (tr)^3$.

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MBMT Pascal Guts Round — Set 5

April 1, 2017

___________ **21** **[9]** For all integers $k > 1$,

$$\sum_{n=0}^{\infty} k^{-n} = \frac{k}{k-1}$$

There exists a sequence of integers $j_0, j_1, \ldots$ such that

$$\sum_{n=0}^{\infty} j_n k^{-n} = \left(\frac{k}{k-1}\right)^3$$

for all integers $k > 1$. Find j_{10}.

___________ **22** **[9]** Nimi is a triangle with vertices located at $(-1, 6)$, $(6, 3)$, and $(7, 9)$. His center of mass is tied to his owner, who is asleep at $(0, 0)$, using a rod. Nimi is capable of spinning around his center of mass and revolving about his owner. What is the maximum area that Nimi can sweep through?

___________ **23** **[9]** The polynomial $x^{19} - x - 2$ has 19 distinct roots. Let these roots be $\alpha_1, \alpha_2, \ldots, \alpha_{19}$. Find $\alpha_1^{37} + \alpha_2^{37} + \cdots + \alpha_{19}^{37}$.

___________ **24** **[9]** I start with a positive integer n. Every turn, if n is even, I replace n with $\frac{n}{2}$, otherwise I replace n with $n - 1$. Let k be the most turns required for a number $n < 500$ to be reduced to 1. How many values of $n < 500$ require k turns to be reduced to 1?

___________ **25** **[9]** In triangle ABC, $AB = 13$, $BC = 14$, and $AC = 15$. Let I and O be the incircle and circumcircle of ABC, respectively. The altitude from A intersects I at points P and Q, and O at point R, such that Q lies between P and R. Find PR.

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MBMT Pascal Guts Round — Set 6

April 1, 2017

__________ **26** **[12]** Submit a decimal n to the nearest thousandth between 0 and 200. Your score will be $\min(12, S)$, where S is the non-negative difference between n and the largest number less than or equal to n chosen by another team (if you choose the smallest number, $S = n$). For example, 1.414 is an acceptable answer, while $\sqrt{2}$ and 1.4142 are not.

__________ **27** **[12]** Guang is going hard on his YNA project. From 1:00 AM Saturday to 1:00 AM Sunday, the probability that he is not finished with his project x hours after 1:00 AM on Saturday is $\frac{1}{x+1}$. If Guang does not finish by 1:00 AM on Sunday, he will stop procrastinating and finish the project immediately. Find the expected number of minutes A it will take for him to finish his project.

An estimate of E will earn $12 \cdot 2^{-|E-A|/60}$ points.

__________ **28** **[12]** All the diagonals of a regular 100-gon (a regular polygon with 100 sides) are drawn. Let A be the number of distinct intersection points between all the diagonals. Find A.

An estimate of E will earn $12 \cdot (16 \log_{10}(\max(\frac{E}{A}, \frac{A}{E})) + 1)^{-\frac{1}{2}}$ or 0 points if this expression is undefined.

__________ **29** **[12]** Find the smallest positive integer A such that the following is true: if every integer $1, 2, \ldots, A$ is colored either red or blue, then no matter how they are colored, there are always 6 integers among them forming an increasing arithmetic progression that are all colored the same color.

An estimate of E will earn $12 \min(\frac{E}{A}, \frac{A}{E})$ points or 0 points if this expression is undefined.

__________ **30** **[12]** For all integers $n \geq 2$, let $f(n)$ denote the smallest prime factor of n. Find

$$A = \sum_{n=2}^{10^6} f(n).$$

In other words, take the smallest prime factor of every integer from 2 to 10^6 and sum them all up to get A.

You may find the following values helpful: there are 78498 primes below 10^6, 9592 primes below 10^5, 1229 primes below 10^4, and 168 primes below 10^3.

An estimate of E will earn $\max(0, 12 - 4\log_{10}(\max(\frac{E}{A}, \frac{A}{E})))$ or 0 points if this expression is undefined.