MBMT Counting and Probability Round — Pascal

April 1, 2017

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered by fewer competitors will be weighted more heavily. Please write your answers in the simplest possible form.

_____ **1** 4 people are at a party, and every pair of people shakes hands with each other. How many total handshakes are there?

_____ **2** A palindrome is a nonnegative integer that reads the same forwards and backwards. For example, 12321 and 0 are both palindromes. Find the number of palindromes between 1000 and 9999, inclusive.

_____ **3** Stan has five friends: Allen, Brian, Catherine, Daniel, and Evan. Each of the 6 people took a test and the teacher told each of them their own score privately. Now they want to share their scores with each other. In a conversation, Person A tells Person B all the scores they know, but not vice versa. What is the minimum number of conversations required for every person to know all six scores?

_____ **4** Guang is listening to an album that consists of 21 songs. 13 of the songs are 4 minutes long, 7 of the songs are 1 minute long skits, and one song is 12 minutes long. If he shuffles the songs randomly, what is the probability that the first 3 songs played last a total of at least 9 minutes?

_____ **5** A jar with N candies contains only mints and lollipops, all of which are distinguishable. Mindy wants to take some, but not all, of the mints. Lolly wants to take some, but not all, of the lollipops. Let M be the number of ways in which Mindy can choose her mints, and let L be the number of ways in which Lolly can choose her lollipops. If $M - L = 3840$, what is N?

_____ **6** There are 10 vans in a row, and Daniel paints each of them either white or black, independently and with equal probability. What is the probability that after painting all 10 vans, there will be two vans next to each other that are both white?

_____ **7** Let S be $\{1, 2, 3, ..., 2017\}$. How many ordered 1000-tuples of sets $(X_1, X_2, ..., X_{1000})$ are there such that $X_1 \subseteq X_2 \subseteq ... \subseteq X_{1000} \subseteq S$?

_____ **8** Each point in a 4 by 6 rectangular grid of lattice points is colored either red or blue. A coloring is "good" if it does not contain 4 points of the same color that form the vertices of a rectangle with edges parallel to the grid axes. How many "good" colorings of the 4 by 6 grid are there?