MBMT Algebra Round – Pascal

April 1, 2017

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of 8 questions. You will have 30 minutes to complete the round. Each question is *not* worth the same number of points. Questions answered by fewer competitors will be weighted more heavily. Please write your answers in the simplest possible form.

- 1 Mr. Street is using his newly patented stapling technique to staple packets for his class. If there are 1000 sheets of paper when Mr. Street starts stapling, each packet contains 5 sheets of paper, and each packet takes 2 seconds to staple, how long in seconds does it take Mr. Street to staple all the packets for his class?
 - 2 Kanye West really likes driving from his house to the studio in his Mercedes. On a day where traffic is moving smoothly, Kanye can get to the studio from his house in 30 minutes. On a day with a lot of traffic, it takes Kanye 50 minutes to reach the studio from his house, and his average speed is 20 miles per hour slower than usual. How far, in miles, is the studio from Kanye's house?
 - **3** Alex is playing with some goo. The goo is very magical: each morning at 7:00 AM it will make itself 60% longer. Every day, Alex wakes up at 8:00 AM, plays with it and stretches it by another 25%. If the goo is 1 meter long at 6:59 AM on Monday, on what day of the week will it surpass 2017 meters in length?
 - **4** Let $P(x) = x^4 24x^2 + 36$ with roots a, b, c and d such that a > b > c > d. Find ab + cd.
 - **5** Andrew has n muffins, where n is a positive integer, arranged in an a by b rectangular array. He eats all the muffins in the top row and all the muffins in the left column and then realizes that he has eaten half of the muffins. Compute n.
 - **6** Let S be the set of points (x, y) such that
 - x and y are integers
 - (x, y) is not the origin
 - $0 \le x \le 10$ and $0 \le y \le 10$

For every point P in S, the circle with diameter OP, where O is the origin, is drawn. What is the total area of all the circles drawn?

7 For each positive integer k, Tsew Eynak writes down the number k k times, resulting in the sequence $\{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots\}$. Let f(n) be the average of all the numbers through the nth number in Tsew's sequence. What is the value of n for which $f(n) = \frac{46}{3}$?

8 Let $f(x) = x^3 + 3x^2 + 5x + \frac{1}{2}$ have roots r_1, r_2, r_3 . Compute

$$\left(r_1 + r_2 + \frac{1}{r_1 r_2}\right)\left(r_2 + r_3 + \frac{1}{r_2 r_3}\right)\left(r_3 + r_1 + \frac{1}{r_3 r_1}\right)$$