MBMT Team Round — Lobachevsky Answers

1. What is the sum of the positive divisors of 100?
   Answer: 217

2. Raymond wants to travel in a car with 3 other (distinguishable) people. The car has 5 seats: a driver's seat, a passenger seat, and a row of 3 seats behind them. If Raymond's cello must be in a seat next to him, and he can't drive, but every other person can, how many ways can everyone sit in the car?
   Answer: 24

3. Peter wants to make fruit punch. He has orange juice (100% orange juice), tropical mix (25% orange juice, 75% pineapple juice), and cherry juice (100% cherry juice). If he wants his final mix to have 50% orange juice, 10% cherry juice, and 40% pineapple juice, in what ratios should he mix the 3 juices? Please write your answer in the form (orange):(tropical):(cherry), where the three integers are relatively prime.
   Answer: $11 : 16 : 3$

4. Points A, B, C, and D are chosen on a circle such that $m\angle ACD = 85°$, $m\angle ADC = 40°$, and $m\angle BCD = 60°$. What is $m\angle CBD$?
   Answer: 55

5. Circles A and B are drawn on a plane such that they intersect at two points. The centers of the two circles and the two intersection points lie on another circle, circle C. If the distance between the centers of circles A and B is 20 and the radius of circle A is 16, what is the radius of circle B?
   Answer: 12

6. For how many integers n is $n^2 + 4$ divisible by $n + 2$?
   Answer: 8

7. A small sphere of radius 1 is sitting on the ground externally tangent to a larger sphere, also sitting on the ground. If the line connecting the spheres' centers makes a 60° angle with the ground, what is the radius of the larger sphere?
   Answer: $7 + 4\sqrt{3}$

8. A classroom has 12 chairs in a row and 5 distinguishable students. The teacher wants to position the students in the seats in such a way that there is at least one empty chair between any two students. In how many ways can the teacher do this?
   Answer: 6720

9. Let there be real numbers a and b such that $a/b^2 + b/a^2 = 72$ and $ab = 3$. Find the value of $a^2 + b^2$.
   Answer: 75

10. Find the number of ordered pairs of positive integers (x, y) such that $\gcd(x, y) + \operatorname{lcm}(x, y) = x + y + 8$.
Answer: 8

11. Evaluate $\displaystyle\sum_{i=1}^{\infty} \frac{i}{4^i} = \frac{1}{4} + \frac{2}{16} + \frac{3}{64} + \dots$
Answer: $\frac{4}{9}$

12. Xavier and Olivia are playing tic-tac-toe. Xavier goes first. How many ways can the game play out such that Olivia wins on her third move? The order of the moves matters.
Answer: 5328

13. Let $ABCD$ be a convex quadrilateral with $AC = 20$. Furthermore, let M, N, P, and Q be the midpoints of DA, AB, BC, and CD, respectively. Let X be the intersection of the diagonals of quadrilateral $MNPQ$. Given that $NX = 12$ and $XP = 10$, compute the area of $ABCD$.
Answer: 384

14. Evaluate $(\sqrt{3} + \sqrt{5})^6$ to the nearest integer.
Answer: 3904

15. In Hatland, each citizen wears either a green hat or a blue hat. Furthermore, each citizen belongs to exactly one neighborhood. On average, a green-hatted citizen has 65% of his neighbors wearing green hats, and a blue-hatted citizen has 80% of his neighbors wearing blue hats. Each neighborhood has a different number of total citizens. What is the ratio of green-hatted to blue-hatted citizens in Hatland? (A citizen is his own neighbor.)
Answer: $\frac{4}{7}$