MBMT Sprint Round — Lobachevsky

Full Name ____

Team Number _________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **25** questions. You will have **30** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

1. What is the remainder when the positive integers from 1 to 7 are added together and then divided by 6?

2. What is the units digit of the product of the first 10 primes?

3. What is the smallest possible value of n if $n > 1$ and $(1+2+3+\cdots+n)^2$ is a perfect fourth power?

4. How many terms are in the arithmetic sequence $7, 11, 15, \ldots, 127, 131$?

5. Let $m = \frac{103!}{100!}$. Find the sum of the prime factors of m.

6. Let $f(x) = 2x$ and let $g(x) = 3x - 3$. Find x such that $g(f(x)) = x$.

7. A sphere is intersected with a regular tetrahedron. What is the maximum number of intersection points which lie on the edges of the regular tetrahedron?

8. At the restaurant Seyepop, there is one item on the menu fried chicken. Fried chicken comes in packs of $2, 4, 6, 8, \ldots$ (any even integer) and 15. What is the maximum number of pieces of fried chicken that cannot be purchased using these packs at this Seyepop organization?

9. How many ordered pairs of integers (x, y) satisfy the equation $4x^2 - y^2 + 1 = 0$?

______________ 10. How many of the first 2016 triangular numbers are odd? We define the nth triangular number to be $1 + 2 + 3 + \ldots + n$, where n is a positive integer.

______________ 11. Find the remainder when 2016^{2016} is divided by 31.

______________ 12. Let there be chords AB and CD in circle O such that AB and CD intersect at a point P inside of O. If $AP = 8$, $BP = 9$ and $CP = 6$, find the value of DP.

______________ 13. Evaluate $\sqrt{90 + \sqrt{90 + \sqrt{90 + \ldots}}}$.

______________ 14. Circle O has radius 1. Points A and B on circle O are chosen such that $m\angle AOB = 120°$. What is the area of the smaller region bound by the circle and segment AB?

______________ 15. Alice and Bob are playing a game. Alice rolls 4 6-sided dice and finds the sum, while Bob rolls a single n-sided die. If they get the same number, they both roll again until someone's result is higher. If both people have an equal chance of winning, what is n?

______________ 16. When a positive integer x is divided by 2, the remainder is 1. When x is divided by 3, the remainder is 2. When x is divided by 5, the remainder is 4. When x is divided by 7, the remainder is 6. When x is divided by 11, the remainder is 10. What is the smallest possible value of x?

______________ 17. There are 6 balls in a bag. 2 are red, 2 are blue, and 2 are green. 4 balls are chosen at random, without replacement. What is the probability that there will be at least one of each color?

______________ 18. Mr. Lodal is on a field trip star gazing with his earth science class. Every star is a regular star with $2n$ vertices, which is created by taking the sides of a regular n-gon ($n \geq 5$) and extending the sides until two originally non-adjacent sides meet. Mr. Lodal determines that in a certain type of star, the non-reflex angles are equal to 135 degrees. Determine the number of vertices in this type of star.

______________ 19. Evaluate $162^3 - 3 * 162^2 * 160 + 3 * 160^2 * 162 - 160^3$.

______________ 20. How many 3-digit positive integers have the property that the hundreds digit is strictly the greatest of all the digits?

______________ 21. What is the height of a regular tetrahedron with side length 2?

______________ 22. Let m denote the minimum number of distinct lines necessary to cut the plane into 2016 sections and n denote the maximum number of distinct lines that can cut the plane into exactly 2016 sections. Compute $n - m$.

______________ 23. Cire and Ymerej are playing a game where they alternate turns. The integers from 1 to 120 are placed in a hat and are to be drawn with replacement. Cire wins if he draws a multiple of 2, and Ymerej wins if he draws a multiple of 3. If neither player wins on their turn, the game continues. If Cire goes first, what is the probability that Ymerej wins?

______________ 24. If $a + b + c = 1$ and $a^3 + b^3 + c^3 = \frac{1}{6}$, find the value of $(a + b)(a + c)(b + c)$.

______________ 25. How many ways can 3 characters be chosen from "$AAABBBCCCDDEEFFGHI$" if the characters representing each letter are indistinguishable, each character may only be used once, and order matters? Two such ways are "AAB" and "BAA".