MBMT Number Theory Round — Lobachevsky Answers

1. Alex has a dumpling cart with n dumplings. He is going to a potluck with either 4, 5, or 7 other people. Given that n is the smallest positive integer number of dumplings such that everyone at the potluck, including himself, can get an equal number of dumplings with none left over, find n.
   Answer: 120

2. For how many positive integers n with $n \leq 2016$ is it true that $2016n$ is a perfect square?
   Answer: 12

3. What is the 200th positive integer that is not a multiple of 2, 3, or 5?
   Answer: 749

4. Let $f(n) = n^2 + 2n$ for all positive integers n. Find the greatest possible value of $\gcd(f(n), f(n+1))$ across all positive integers n.
   Answer: 3

5. What is the size of the largest set $S = \{s_1, s_2, \ldots, s_n\}$ such that every pair of elements in S is coprime, none of the elements of S are prime, and $1 < s_i \leq 2016$?
   Answer: 14

6. How many positive divisors does $2^{18} + 2^{10} + 1$ have?
   Answer: 21

7. If $10^k + 1$ divides $10^{100} + 1$ and k is a nonnegative integer, what is the sum of all the possible values of k?
   Answer: 124

8. Let $x = \underbrace{20162016\ldots20162016}_{\text{2016 repeated 2016 times}}$ in some base b ($b > 1$). If $b - 1 | x$, how many possible values of b are there?
   Answer: 56