MBMT Number Theory Round — Lobachevsky

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is **not** worth the same number of points. Questions answered by fewer competitors are weighted more heavily. Please write your answers in the simplest possible form.

1. Alex has a dumpling cart with n dumplings. He is going to a potluck with either 4, 5, or 7 other people. Given that n is the smallest positive integer number of dumplings such that everyone at the potluck, including himself, can get an equal number of dumplings with none left over, find n.

2. For how many positive integers n with $n \leq 2016$ is it true that $2016n$ is a perfect square?

3. What is the 200th positive integer that is not a multiple of 2, 3, or 5?

4. Let $f(n) = n^2 + 2n$ for all positive integers n. Find the greatest possible value of $\gcd(f(n), f(n+1))$ across all positive integers n.

5. What is the size of the largest set $S = \{s_1, s_2, \ldots, s_n\}$ such that every pair of elements in S is coprime, none of the elements of S are prime, and $1 < s_i \leq 2016$?

6. How many positive divisors does $2^{18} + 2^{10} + 1$ have?

7. If $10^k + 1$ divides $10^{100} + 1$ and k is a nonnegative integer, what is the sum of all the possible values of k?

8. Let $x = \underbrace{20162016\ldots20162016}_{\text{2016 repeated 2016 times}}$ in some base b ($b > 1$). If $b - 1 | x$, how many possible values of b are there?