

MBMT Geometry Round — Lobachevsky

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is **not** worth the same number of points. Questions answered by fewer competitors are weighted more heavily. Please write your answers in the simplest possible form.

1. Let $ABCD$ be a square with side length 30. A circle centered at the center of $ABCD$ with diameter 34 is drawn. Let E and F be the points at which the circle intersects side AB . What is EF ?
2. What is the area of the quadrilateral bounded by $|2x| + |3y| = 6$?
3. Right triangle ABC has hypotenuse AB . Altitude CD divides AB into segments AD and DB , with $AD = 20$ and $DB = 16$. What is the area of triangle ABC ?
4. Circle O has chord AB . Extend AB past B to a point C . A ray from C is drawn, and this ray intersects circle O . Let point D be the point of intersection of the ray and the circle that is closest to point C . Given $AB = 20$, $BC = 16$, and $OA = \frac{201}{6}$, find the longest possible length of CD .
5. Consider a circular cone with vertex A . The cone's height is 4 and the radius of its base is 3. Inscribe a sphere inside the cone. Find the ratio of the volume of the cone to the volume of the sphere.
6. A disk of radius $\frac{1}{2}$ is randomly placed on the coordinate plane. What is the probability that it contains a lattice point (point with integer coordinates)?
7. Let ABC be an equilateral triangle of side length 2. Let D be the midpoint of BC , and let P be a variable point on AC . By moving P along AC , what is the minimum perimeter of triangle BDP ?
8. Let $ABCD$ be a rectangle with $AB = 8$ and $BC = 9$. Let $DEFG$ be a rhombus, where G is on line BC and A is on line EF . If $m\angle EFG = 30^\circ$, what is DE ?