MBMT Algebra Round — Lobachevsky Answers

1. What is the largest integer n for which $n^{24} < 20^{16}$?
Answer: 7

2. The sequence E_n has the property that $E_1 = 20$ and $E_n = E_{n-2} + E_{n-1}$ for all $n \geq 3$. If $E_5 = 16$, what is E_7?
Answer: 36

3. Let $f(x) = x^3 + 1729x^2 + 1728x + 1727$. Find the sum of the roots of $f(x+2)$.
Answer: -1735

4. Danny and Jason each choose a positive integer. They notice that Danny's integer, Jason's integer, and their product minus 4 times Danny's integer form an arithmetic sequence, in that order. Let a be Danny's integer and b be Jason's integer. What are all possible ordered pairs (a, b)?
Answer: $(3, 9), (4, 6), (5, 5), (8, 4)$

5. Compute $50 * 50 + 51 * 49 + 52 * 48 + \ldots + 99 * 1 + 100 * 0$.
Answer: 84575

6. $f(x)$ has the property that $5f(x) - 3f\left(\frac{1}{x}\right) = x^3$ for all nonzero x. Find $f(\sqrt[3]{6})$.
Answer: $\frac{61}{32}$

7. Evaluate $\displaystyle\sum_{i=0}^{\infty} \frac{2}{(n+1)(n+5)} = \frac{2}{1*5} + \frac{2}{2*6} + \frac{2}{3*7} + \ldots$
Answer: $\frac{25}{24}$

8. Let $f(x)$ be a function such that $f(x)f(y) - f(xy) = xy$ for all real x and y. Let M and m be the maximum possible value and minimum possible value, respectively, of $f(2016)$. Find $M - m$.
Answer: $2016\sqrt{5}$