

Team # _____

MBMT Guts Round Set 1

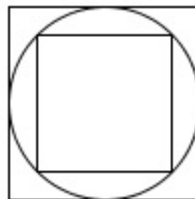
_____ 1. Mr. Stein is ordering a two-course dessert at a restaurant. For each course, he can choose to eat pie, cake, rødgrød, and crème brûlée, but he doesn't want to have the same dessert twice. In how many ways can Mr. Stein order his meal? (Order matters.)

_____ 2. Evaluate $\frac{1}{2 + \frac{3}{1 + \frac{2}{3+x}}}$ if $x = 1$.

_____ 3. A positive integer n is divisible by 3 and 5, but not by 2. If $n > 20$, what is the smallest possible value of n ?

_____ 4. Find the fourth-smallest positive integer that can be expressed as the product of two different prime numbers.

_____ 5. In the diagram below, the larger square has side length 6. Find the area of the smaller square.



MBMT Guts Round Set 2

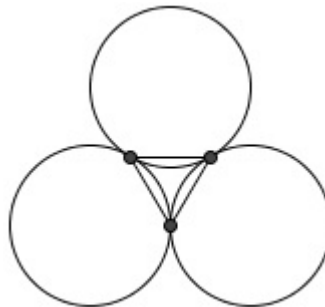
_____ 6. A regular n -gon has diagonals that all have the same length. What is the maximum possible value of n ?

_____ 7. If $x + y = 306$ and $\frac{x}{y} = \frac{7}{10}$, compute $y - x$.

_____ 8. The school store is running out of supplies, but it still has five items: one pencil (costing \$1), one pen (costing \$1), one folder (costing \$2), one pack of paper (costing \$3), and one binder (costing \$4). If you have \$10, in how many ways can you spend your money? (You don't have to spend all of your money, or any of it.)

_____ 9. Alice and Bob are builders; Charlie is a destroyer. Alice can build a car in 20 hours and Bob can build a car in 10 hours, while Charlie destroys a car in 40 hours. If Alice and Bob are working together on a car Charlie is destroying, how many hours will it take for Alice and Bob to finish building the car?

_____ 10. Three circles of radius 1 are mutually tangent, as shown. What is the area of the triangle whose vertices are the points of tangency?



MBMT Guts Round Set 3

- _____ 11. In the middle of the school year, 40% of Poolesville magnet students decided to transfer to the Blair magnet, and 5% of the original Blair magnet students transferred to the Poolesville magnet. If the Blair magnet grew from 400 students to 480 students, how many students does the Poolesville magnet have after the transferring has occurred?
- _____ 12. A square with side length 6 is rotated by 90° about its center. What is the area of the region swept out by the perimeter of the square (that is, the four line segments forming the boundary of the square)?
- _____ 13. A bag contains ten red marbles and some number of blue marbles. If two marbles are chosen without replacement, the probability that they are both red is $\frac{5}{17}$. How many marbles are in the bag?
- _____ 14. What number is nine more than four times the answer to this question?
- _____ 15. Two (not necessarily different) numbers are chosen independently and at random from $\{1, 2, 3, \dots, 10\}$. On average, what is the product of the two integers? (Compute the expected product. That is, if you do this over and over again, what will the product of the integers be on average?)

MBMT Guts Round Set 4

_____ 16. Your math teacher asks you to rationalize the denominator of the expression $\frac{a}{b+\sqrt{c}}$, where a , b , and c are integers and c is not divisible by the square of any prime. You find that $\frac{a}{b+\sqrt{c}}$ is equal to $\frac{30-5\sqrt{14}}{11}$. Compute the triple (a, b, c) .

_____ 17. Let G, O, D, I, and T be digits that satisfy the following equation:

$$\begin{array}{r} \text{GOGO} \\ +\text{DIDI} \\ \hline \text{GODOT} \end{array}$$

(Note that G and D cannot be 0, and that the five variables are not necessarily different.)

Compute the value of GODOT.

_____ 18. The first triangle number is 1; the second is $1 + 2 = 3$; the third is $1 + 2 + 3 = 6$; and so on. Find the sum of the first 100 triangle numbers.

_____ 19. A checkerboard is 91 squares long and 28 squares wide. A line connecting two opposite vertices of the checkerboard is drawn. How many squares does the line pass through?

_____ 20. How many lattice points are exactly twice as close to $(0, 0)$ as they are to $(15, 0)$? (A lattice point is a point (a, b) such that both a and b are integers.)

MBMT Guts Round Set 5

- _____ 21. A bug starts at vertex A of triangle ABC . Six times, the bug travels to a randomly chosen adjacent vertex. For example, the bug could go from A , to B , to C , back to B , and back to C . What is the probability that the bug ends up at A after its six moves?
- _____ 22. In rhombus $ABCD$, $\angle A = 60^\circ$. Rhombus $BEFG$ is constructed, where E and G are the midpoints of BC and AB , respectively. Rhombus $BHIJ$ is constructed, where H and J are the midpoints of BE and BG , respectively. This process is repeated forever. If the area of $ABCD$ and the sum of the areas of all of the rhombi are both integers, compute the smallest possible value of AB .
- _____ 23. A positive integer is called *oneic* if it consists of only 1's. For example, the smallest three oneic numbers are 1, 11, and 111. Find the number of 1's in the smallest oneic number that is divisible by 63.
- _____ 24. In cyclic quadrilateral $ABCD$, $\angle DBC = 90^\circ$ and $\angle CAB = 30^\circ$. The diagonals of $ABCD$ meet at E . If $\frac{BE}{ED} = 2$ and $CD = 60$, compute AD . (Note: a cyclic quadrilateral is a quadrilateral that can be inscribed in a circle.)
- _____ 25. Three real numbers a , b , and c between 0 and 1 are chosen independently and at random. What is the probability that $a + 2b + 3c > 5$?

MBMT Guts Round Set 6: Estimation

- _____ 26. Choose a real number between 0 and 10, inclusive. If your number is less than the average of all numbers chosen, you will get your number's worth of points, but if your number is greater than or equal to the average, you will get 0 points. For example, if the average of all numbers chosen is 1.2, and you pick 1.6, then you will receive 0 points, but if you pick 0.5, then you will receive 0.5 points. Express your answer to the nearest thousandth. For example, 7.800, 2.110, and 0.234 are valid responses, but 7.8 and 0.2345 are not. An invalid response will receive a score of zero.
- _____ 27. There are 1000 balls and 500 bins that can fit arbitrarily many balls. All of the balls are then placed independently and at random into the bins. Estimate how many bins, on average, are empty. (Estimate the expected number of empty bins. In other words, if this were done over and over again, how many bins would be empty on average?) Your estimate must be an integer, or you will receive a score of zero.
- _____ 28. Estimate the smallest value of r such that a circle of radius r can contain 19 non-overlapping circles of radius 1. Express your answer to the nearest hundredth. For example, 11.00, 5.60, and 1.34 are valid responses, but 11 and 1.342 are not. An invalid response will receive a score of zero.
- _____ 29. A fair coin is flipped 2015 times. Estimate the probability that fewer than 1000 heads are flipped. Express your answer to the nearest thousandth. For example, 0.800, 0.110, and 0.234 are valid responses, but 0.8 and 0.2345 are not. An invalid response will receive a score of zero.
- _____ 30. Estimate the number of positive integers less than or equal to 1,000,000 that can be expressed as the sum of two nonnegative integer squares. Your estimate must be an integer, or you will receive a zero.