MBMT Team Round — Fermat

Team Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

DO NOT TURN THIS TEST IN! You will turn in your answers on the official answer sheet.

This is the most difficult round of the competition. For this reason, you are encouraged to work with your teammates on the problems in order to solve them.

1. Point A is located at $(0, 0)$. Point B is located at $(2, 3)$ and is the midpoint of AC. Point D is located at $(10, 0)$. What are the coordinates of the midpoint of segment CD?

2. Mr. Rose gave a test to his two calculus classes. His first period class has 20 students, and their average score on the test was 80. His second period class has 30 students, and their average score was 90. What was the average score of all 50 of his calculus students?

3. Compute $1 - 2 + 3 - 4 + \cdots + 2013 - 2014 + 2015$.

4. For a list of 9 positive integers that are not necessarily all different, the mean, median, and (unique) mode are all 9. What is largest possible positive difference between the largest element and the smallest element of the set?

5. An unfair 6-sided die has faces labeled 1, 2, 3, 4, 5, and 6. The probability that a die lands with a certain face up is proportional to the number on the face. What is the probability that at least one of the first three rolls is a 1 or a 2?

______ 6. If a, b and c are real numbers such that $ab = 31$, $ac = 13$, and $bc = 5$, compute the product of all possible values of abc.

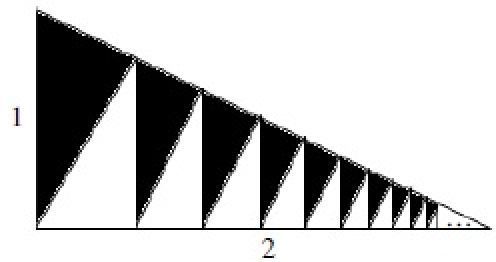
______ 7. Compute $\frac{x^2+8x+7}{x^2+9x+14}$, if $x = 2015$.

______ 8. Victor has 3 piles of 3 cards each. He draws all of the cards, but cannot draw a card until all the cards above it have been drawn. (For example, for his first card, Victor must draw the top card from one of the 3 piles.) In how many orders can Victor draw the cards?

______ 9. If $x + y = 3$ and $x^2 + y^2 = 7$, compute $x^3 + y^3 + x^4 + y^4$.

______ 10. Mr. Rose, Mr. Stein, and Mr. Schwartz start at the same point around a circular track and run clockwise. Mr. Stein completes each lap in 6 minutes, Mr. Rose in 10 minutes, and Mr. Schwartz in 18 minutes. How many minutes after the start of the race are the runners at identical points around the track (that is, they are aligned and are on the same side of the track) for the first time?

11. The right triangle below has legs of length 1 and 2. Find the sum of the areas of the shaded regions (of which there are infinitely many), given that the regions into which the triangle has been divided are all right triangles.



12. Let ABC be a triangle with $AB = 13$, $BC = 14$, and $AC = 15$. Let D and E be the midpoints of BC and AB, respectively. If AD and CE intersect at G, compute the area of quadrilateral $BEGD$.

13. Two (not necessarily different) integers between 1 and 60, inclusive, are chosen independently and at random. What is the probability that their product is a multiple of 60?

14. Let $ABCD$ be a square with side length 1. If point E is on BC, point F is on DC, and triangle AEF is equilateral, compute the side length of triangle AEF. (Note: if your answer has a square root inside a square root, you have not fully simplified your answer.)

15. Adam, Bendeguz, Cathy, and Dennis all see a positive integer n. Adam says, "n leaves a remainder of 2 when divided by 3." Bendeguz says, "For some k, n is the sum of the first k positive integers." Cathy says, "Let s be the largest perfect square that is less than $2n$. Then $2n - s = 20$." Dennis says, "For some m, if I have m marbles, there are n ways to choose two of them." If exactly one of them is lying, what is n?