

## MBMT Number Theory Round — Fermat

Full Name \_\_\_\_\_

Team Number \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

- \_\_\_\_\_ 1. How many integers between 100 and 200, inclusive, are multiples of 3?
  
- \_\_\_\_\_ 2. I have a positive integer number of pets. One-third of them are cats, one-fourth are dogs, and one-eighth are fish. What is the minimum number of pets I could have?
  
- \_\_\_\_\_ 3. What is  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{4}{5}$  of  $\frac{5}{6}$  of  $\frac{6}{7}$  of  $\frac{7}{8}$  of  $\frac{8}{9}$  of  $\frac{9}{10}$  of 1000?
  
- \_\_\_\_\_ 4. Compute the last digit of  $1! + 2! + 3! + 4! + 5! + \cdots + 10!$ .
  
- \_\_\_\_\_ 5. What is the 100th digit to the right of the decimal point in the decimal representation of  $\frac{23}{99}$ ?
  
- \_\_\_\_\_ 6. A science teacher is opening his students' lockers, which are in a row and are numbered 1 through 100 in order. He opens every other locker, starting from locker 1 (so he opens lockers 1, 3, 5, and so on). Then he goes back to the beginning and opens every other unopened locker (so he opens 2, 6, 10, and so on). He continues this until all lockers are open. What is the number of the last locker he opens?
  
- \_\_\_\_\_ 7.  $4004 = 44 \cdot 91$ , so 4004 is a multiple of the number obtained by removing its middle two digits. How many 4-digit palindromes abba are divisible by the 2-digit palindrome aa?
  
- \_\_\_\_\_ 8. Bob and his 30 friends are sharing a huge plate of cookies. When they try to split the cookies evenly, there are 17 cookies left over. In an effort to create an even split, Bob volunteers not to receive any cookies. However, Bob's 30 friends find that if they try to split the cookies evenly, there are 16 cookies left over. What is the smallest number of cookies that could be on the plate?