MBMT Team Round — Euler

Team Name ____

Team Number __________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

DO NOT TURN THIS TEST IN! You will turn in your answers on the official answer sheet.

This is the most difficult round of the competition. For this reason, you are encouraged to work with your teammates on the problems in order to solve them.

1. Compute $1 - 2 + 3 - 4 + \cdots + 2013 - 2014 + 2015$.

2. An unfair 6-sided die has faces labeled 1, 2, 3, 4, 5, and 6. The probability that a die lands with a certain face up is proportional to the number on the face. What is the probability that at least one of the first three rolls is a 1 or a 2?

3. If a, b and c are real numbers such that $ab = 31$, $ac = 13$, and $bc = 5$, compute the product of all possible values of abc.

4. Compute $\frac{x^2+8x+7}{x^2+9x+14}$, if $x = 2015$.

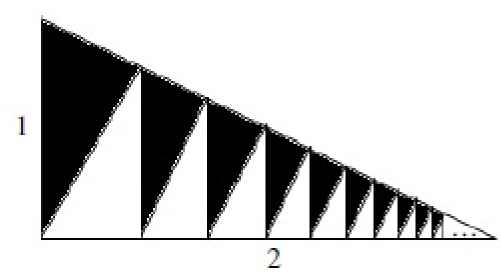
5. Victor has 3 piles of 3 cards each. He draws all of the cards, but cannot draw a card until all the cards above it have been drawn. (For example, for his first card, Victor must draw the top card from one of the 3 piles.) In how many orders can Victor draw the cards?

______ 6. If $x + y = 3$ and $x^2 + y^2 = 7$, compute $x^3 + y^3 + x^4 + y^4$.

______ 7. Mr. Rose, Mr. Stein, and Mr. Schwartz start at the same point around a circular track and run clockwise. Mr. Stein completes each lap in 6 minutes, Mr. Rose in 10 minutes, and Mr. Schwartz in 18 minutes. How many minutes after the start of the race are the runners at identical points around the track (that is, they are aligned and are on the same side of the track) for the first time?

______ 8. You are trying to maximize a function of the form $f(x, y, z) = ax + by + cz$, where a, b, and c are constants. You know that $f(3, 1, 1) > f(2, 1, 1)$, $f(2, 2, 3) > f(2, 3, 4)$, and $f(3, 3, 4) > f(3, 3, 3)$. For $-5 \leq x, y, z \leq 5$, what value of (x, y, z) maximizes the value of $f(x, y, z)$? Give your answer as an ordered triple.

______ 9. The right triangle below has legs of length 1 and 2. Find the sum of the areas of the shaded regions (of which there are infinitely many), given that the regions into which the triangle has been divided are all right triangles.



______ 10. Let ABC be a triangle with $AB = 13$, $BC = 14$, and $AC = 15$. Let D and E be the midpoints of BC and AB, respectively. If AD and CE intersect at G, compute the area of quadrilateral $BEGD$.

11. Two (not necessarily different) integers between 1 and 60, inclusive, are chosen independently and at random. What is the probability that their product is a multiple of 60?

12. Adam, Bendeguz, Cathy, and Dennis all see a positive integer n. Adam says, "n leaves a remainder of 2 when divided by 3." Bendeguz says, "For some k, n is the sum of the first k positive integers." Cathy says, "Let s be the largest perfect square that is less than $2n$. Then $2n - s = 20$." Dennis says, "For some m, if I have m marbles, there are n ways to choose two of them." If exactly one of them is lying, what is n?

13. A blind ant is walking on the coordinate plane. It is trying to reach an anthill, placed at all points where both the x-coordinate and y-coordinate are odd. The ant starts at the origin, and each minute it moves one unit either up, down, to the right, or to the left, each with probability $\frac{1}{4}$. The ant moves 3 times and doesn't reach an anthill during this time. On average, how many additional moves will the ant need to reach an anthill? (Compute the expected number of additional moves needed.)

14. Jane tells you that she is thinking of a three-digit number that is greater than 500 that has exactly 20 positive divisors. If Jane tells you the sum of the positive divisors of her number, you would not be able to figure out her number. If, instead, Jane had told you the sum of the *prime* divisors of her number, then you also would not have been able to figure out her number. What is Jane's number? (Note: the sum of the prime divisors of 12 is $2 + 3 = 5$, not $2 + 2 + 3 = 7$.)

15. In quadrilateral $ABCD$, diagonals AC and BD intersect at O. If the area of triangle DOC is 4 and the area of triangle AOB is 36, compute the minimum possible value of the area of $ABCD$.