MBMT Sprint Round — Euler

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **25** questions. You will have **30** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

_____ 1. A square has perimeter 44. What is the area of the square?

_____ 2. A bag contains 1 red ball, 4 yellow balls, and 5 blue balls. A ball is randomly drawn from the bag. What is the probability that the ball is *not* yellow?

_____ 3. What is the area of a right triangle with one leg of length 15 and a hypotenuse of length 17?

_____ 4. A trapezoid has parallel sides of length 5 and 9 and an area of 42. Compute the height of the trapezoid.

_____ 5. Adam cuts a rectangular cake two times horizontally and three times vertically. How many pieces of cake does he end up with?

_____ 6. If $a = 2b$, $b = 2c$, and $c = 2d$, and $a \neq 0$, compute $\frac{d}{a}$.

_____ 7. What is the smallest positive integer that is divisible by 2, 3, 4, 5, and 6?

8. Three years ago, Alice was twice as old as Bob. Today, the sum of their ages is 30. Find Bob's age today.

9. Johnny is at $(0, 0)$ in Cartesian World and is faced with two choices. He can either go straight to his destination $(12, 9)$, or go to buy a snack at $(12, 5)$ and then go to his destination. How much longer is the path he would have to take if he were to buy the snack? Assume that he takes the shortest path possible in both cases.

10. Compute the product of all solutions to the equation $|x + 5| = 10$.

11. A cube has volume 20. If the lengths of the cube's edges are doubled, what is the volume of the new cube?

12. In a football game, there are two types of scoring events: a touchdown (7 points) and a field goal (3 points). There are two teams in a football game. What is the minimum number of scoring events that must occur for the score difference between the two teams to be exactly 5 points?

13. If x and y are positive integers that add up to 2015, what is the smallest possible value of $\frac{x+y}{x-y}$?

14. Compute the minimum possible value of $x^2 - 2x + y^2 - 4y + 10$.

15. Let M be the region of points on the xy-plane satisfying $|x| + |y| \leq 9$. What is the area of M?

16. A teacher wants to seat three boys and three girls around a circular table so that no one sits next to a person of the same gender. In how many ways can the teacher do this? Remember that all of the students are distinguishable. (Note: two seatings, one of which is simply a rotation of the other, are nevertheless considered different.)

17. Mr. Pham rolls a fair 4-sided die (with the numbers 1, 2, 3, and 4) four times. What is the probability that the numbers he rolls are either all the same or all different?

18. The diagonals of a certain rectangle have length 2 and intersect at a 60° angle. What is the area of the rectangle?

19. What is length of the shortest altitude of a triangle with sides of length 6, 8, and 10?

20. A science teacher has four students in his class — Eric, Mike, Matthew, and Victor — but he can't remember which student has which name. If he randomly calls the four students by the four names so that he uses each name once, what is the probability that he gets at least one name right?

21. Jenny has 5 piles of 3 cards each. She draws 4 cards, but cannot draw a card until all the cards above it have been drawn. (For example, for her first card, Jenny must draw the top card from one of the piles.) If order matters, in how many ways can Jenny draw the cards?

22. A function $f(n)$ is defined as follows: $f(1) = 1$ and $f(2n) = n \cdot f(n)$. If $f(2^{32}) = 2^m$, compute m.

23. Compute the largest integer n such that $\frac{2015!}{13^n}$ is an integer.

24. Compute the remainder when 112358132134 is divided by 66.

25. Vijay the DJ plays songs at the Teejay High School dance for one hour. He has an unlimited number of songs of lengths 1 minute, 2 minutes, 3 minutes, and so on. After each song, he needs a minute to load a new song. If Vijay plays songs to maximize the product of the lengths of the songs he plays in minutes, what is the most common song length he uses, in minutes?