MBMT Algebra Round — Euler

Full Name _____

Team Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is worth the same number of points. Please write your answers in the simplest possible form.

1. Two years ago, Mike put \$100 into a bank. Every year, the amount of money Mike has in his bank account increases by 20%. How many dollars does Mike now have in the bank account?

2. Given that x and y are real numbers such that $x^2 - y^2 = 16$ and $x + y = 2$, compute xy.

3. If $x + y + z = 15$ and $\frac{x}{y+z} = 4$, compute x.

4. Let $a_1 = 1$. For $n > 1$, let $a_n = (n-1)a_{n-1} + (n-2)a_{n-2} + \cdots + 2a_2 + a_1$. Compute a_7.

5. In a certain field, grass grows at a constant rate, no matter the height of the grass. If Farmer John allows 20 cows to eat the grass, it would take them 15 days to eat all the grass in the field. If he allows 30 cows to eat, it would take them 9 days to eat all the grass. How many cows are needed to finish the field in exactly 25 days?

6. If $a^3b^5 = 3072$ and $\frac{b^7}{a^2} = 324$, then $ab^{136} = 2^m 3^n$, for some positive integers m and n. Compute $m + n$.

7. If $x^2 + 17y^2 + 10z^2 = 8xy + 6yz + 2z - 1$, find the ordered triple (x, y, z).

8. Compute the greatest real value of x such that $x^2 - 4x - 3 - \frac{4}{x} + \frac{1}{x^2} = 0$.