MBMT Team Round — Jacobi

March 8, 2026

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **15** questions. You will have **45** minutes to complete the round. Later questions are worth more points; point values are notated next to the problem statement. (There are a total of 100 points.) Please write your answers in the simplest possible form.

DO NOT TURN THE QUESTION SHEET IN!
Use the official answer sheet.

You are highly encouraged to work with your teammates on the problems in order to solve them.

MBMT Team Round Answer Sheet — Jacobi

March 8, 2026

Team Name _____

Team Number _____

1. _____ **9.** _____

2. _____ **10.** _____

3. _____ **11.** _____

4. _____ **12.** _____

5. _____ **13.** _____

6. _____ **14.** _____

7. _____ **15.** _____

8. _____

1. **[4]** Spongebobert Squaretrousers lives in a cube under the sea of side length 2, and Spongerobert Rectanglepants lives in a rectangular prism above the sea with double the height, triple the length, and triple the width of Spongebobert's house. What is the volume of Spongerobert's house?

2. **[4]** Sam flips a fair coin 4 times. What is the probability that he flips the same face twice in a row, and then flips the other face twice in a row?

3. **[4]** Square $ABCD$ has side length 8. What is the area of the square formed by the midpoints of the sides of $ABCD$?

4. **[5]** Let $\lfloor x \rfloor$ denote the greatest integer less than x. What is the unique real number x that satisfies $x \cdot \lfloor x \rfloor = 10$?

5. **[5]** Let a *half-prime* number be a positive integer n such that among the first n positive integers, *exactly* half of them are prime. What is the average of all half-prime numbers?

6. **[5]** Alice, Bill, and Chris each start with the same amount of apple juice in their cups. Alice pours 2/3 of her apple juice into Bill's cup, and afterwards Bill pours 4/5 of his current apple juice into Chris's cup. If they pour all of their juice into one bowl, what fraction of the juice in the bowl was poured from Chris's cup?

7. **[6]** Let $ABCDE$ be a convex pentagon with segment $\overline{BC}$ parallel to segment $\overline{AD}$, $AB = BC = CD = 4$, and $DE = EA = AD = 8$. What is the area of pentagon $ABCDE$?

8. **[6]** Akliu randomly selects a point on the xy-plane inside the square with vertices $(-1, -1)$, $(-1, 1)$, $(1, -1)$, and $(1, 1)$. He then draws a line through that point and the origin. What is the probability his line has a slope greater than 2?

9. [7] Daniel has a 4 inch by 8 inch piece of rectangular origami paper. He first chooses a vertex of the paper, then folds the vertex inward in one step so that the vertex is mapped to the center of the rectangle, and then unfolds the paper, leaving a straight crease in the paper. He then repeats this process for each of the remaining vertices. After this process, 4 creases are left in the paper. What is the area of the hexagon bounded by these 4 creases and the paper?

10. [8] For some positive integer n, let $f(n)$ output the smallest positive integer such that $n + f(n)$ has exactly 20 factors. What is $f(f(f(26)))$?

11. [8] A unit fraction is any simplified fraction with a numerator of 1. What is the sum of all unit fractions whose decimal representations terminate?

12. [9] For some positive integer n, there exists a positive integer d such that n has $6d$ factors, $2n$ has $8d$ factors, and $3n$ has $9d$ factors. What is the sum of all possible integers n for $n \leq 150$?

13. [9] Lewis is standing at the origin of the xy-plane. At any given moment, he has a $1/2$ chance of moving 1 unit upward and a $1/2$ chance of moving 1 unit to the right. Given that he never reaches the point $(3, 3)$, what is the probability that Lewis will reach the point $(4, 4)$?

14. [10] What is the remainder when

$$2026^{2025+2024^{2023+2022^{\cdot^{\cdot^{\cdot^{3+2^1}}}}}}$$

is divided by 101?

15. [10] $\triangle ABC$ is triangle such that the foot of the altitude from A to $\overline{BC}$ is D, and the line AD intersects the circumcircle of $\triangle ABC$ at a point $E \neq A$. Let M be the midpoint of $\overline{BC}$. Given that $BD = 6$, $DC = 7$, and that circumcircle of triangle $\triangle MDE$ is tangent to the circumcircle of $\triangle ABC$, what is the area of triangle $\triangle ABC$?