

MBMT Number Theory Round — Jacobi

March 8, 2026

Full Name _____

Student ID Number _____

**DO NOT BEGIN UNTIL YOU ARE
INSTRUCTED TO DO SO.**

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

- _____ 1. What is the smallest positive perfect square that is divisible by 8?
- _____ 2. What is the sum of the smallest and largest 2-digit integers whose squares end in a 6?
- _____ 3. Kenneth is thinking of a 3-digit number. When he removes the units digit of the number, his new number is divisible by 15. When he removes the hundreds digit of the number, his new number is still divisible by 15. What is the largest possible 3-digit number that Kenneth could be thinking of?
- _____ 4. A semiprime is a number that is the product of two distinct primes. Yunyi takes the product of two semiprimes and finds the resulting number has n factors. What is the sum of all possible values of n ?
- _____ 5. How many positive integer divisors of 20^{26} have an odd number of divisors?
- _____ 6. Let $S(n)$ denote the sum of digits of n . How many positive integers $k < 1000$ exist such that there exists some positive integer n that satisfies $k = n - S(n)$?
- _____ 7. Let $f(n) = \text{lcm}(1, 2, 3, \dots, n)$. For how many positive integers $n < 100$ does $f(n) = f(n + 1)$?
- _____ 8. Define the function $f(n)$ as

$$f(n) := \sum_{i=1}^n \gcd(i, n).$$

Sam takes the product of the first 8 primes and gets a number k . How many factors does $f(k)$ have?