1. Four points A, B, C, and D, lie on a line in some order and satisfy $BC = 4$, $AC = 6$, and $BD = 12$. What is the largest possible value of AD?

Proposed by Sam Easaw.

Answer: $\boxed{22}$

Solution: Start by drawing BD on a line. Then, C must lie within 4 units of B in either direction, and A must lie within 6 units of C in either direction, so we place C and further A as far away from D as possible. This results in a line of points A, C, B, D, or reversed. Thus, $AD = AC + BC + BD = 12 + 6 + 4 = \boxed{22}$.

2. What is the distance from the origin to the point $(3, 4, 12)$?

Proposed by Sam Easaw.

Answer: $\boxed{13}$

Solution: The distance from the origin to a point (a, b, c) is given by $d = \sqrt{a^2 + b^2 + c^2}$. Thus, $d = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{5^2 + 12^2} = \sqrt{13^2} = \boxed{13}$.

3. A rhombus of side length 6 has a diagonal of length 6. What is the area of the rhombus?

Proposed by Sam Easaw.

Answer: $\boxed{18\sqrt{3}}$

Solution: Note that a rhombus has all equal side lengths. If there is a diagonal of length 6, it must be the shorter diagonal by the triangle inequality. The rhombus can now be split into two equilateral triangles, each with side length 6, which each have area $\frac{6^2\sqrt{3}}{4} = 9\sqrt{3}$. Thus, the area of the rhombus is $\boxed{18\sqrt{3}}$.

4. Let P be a hexagon of side length 2. Andy draws a flower-shape onto the hexagon by constructing a circle of radius 1 at each vertex of P. What is the perimeter of Andy's new closed flower?

Proposed by Siyuan Yan.

Answer: $\boxed{8\pi}$

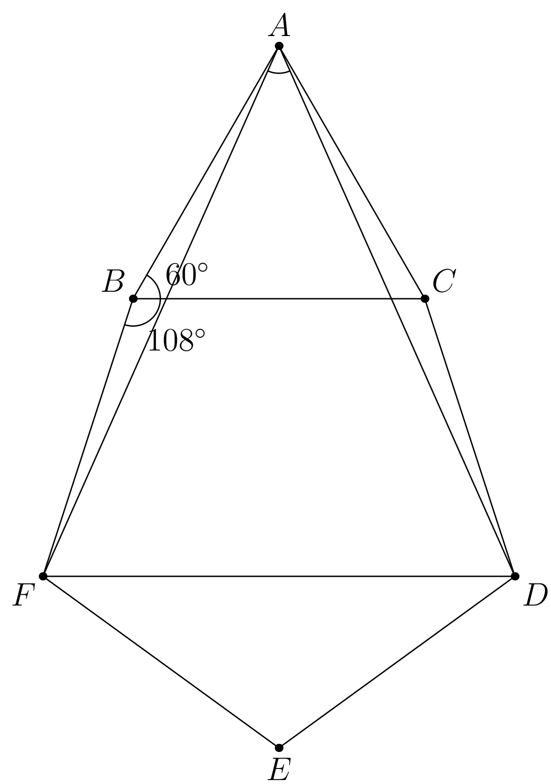
Solution: Since the hexagon has side length 2, and the circles have radius 1, all of the circles must be tangent to each other, so we just need to find the perimeter of the circles that lie outside the hexagon. Since each angle in a hexagon is $120°$, each circle has $120°$ of circumference lying inside of the circle, and thus $360° - 120° = 240°$ of circumference lying outside of it. This is $\frac{240°}{360°} = \frac{2}{3}$ of circumference for each circle. Since there are 6 circles, we have $\frac{2}{3} \cdot 6 = 4$ circumferences of one circle in total. Thus, the answer is $4 \cdot (2 \cdot \pi \cdot 1) = \boxed{8\pi}$.

5. Equilateral triangle $\triangle ABC$ has a side length of 1. A regular pentagon $BCDEF$ is constructed on the exterior of $\triangle ABC$. In degrees, what is the measure of angle $\angle DAF$?

Proposed by Sam Easaw.

Answer: $\boxed{48°}$

Solution:



Refer to above diagram. We know that each interior angle in an equilateral triangle measures $60°$, and in a regular pentagon measures $(5-2) \cdot 180°/5 = 108°$. Note that $\triangle ABF$ is isosceles, and $\angle ABF = 60° + 108° = 168°$, so the base angles each measure $\frac{1}{2}(180° - 168°) = 12°/2 = 6°$. Now, the measure of

$\angle DAF$ is equal to $60^\circ - \angle BAF - \angle CAD = 60^\circ - 2 \cdot \angle BAF$ by symmetry, so $\angle DAF = 60^\circ - 2 \cdot 6^\circ = \boxed{48^\circ}$.

6. Triangle $\triangle ABC$ has $AB = 16$ and $AC = 17$. There exists a point P outside $\triangle ABC$ such that $AP = 16$, $CP = 17$, and $\overline{AC}$ is perpendicular to $\overline{PB}$. What is the area of quadrilateral $PABC$?

Proposed by Sam Easaw.

Answer: $\boxed{240}$

Solution: Note that $PC = AC = 17$, so triangle $\triangle PCA$ is isosceles. Also, $AB = AP = 16$, so $\triangle PBA$ is also isosceles. Now, since $\overline{AC}$ is perpendicular to $\overline{PB}$, we know that $\overline{AC}$ is the altitude of $\triangle PAB$, which is also the angle bisector of $\angle BAP$ since $\triangle PAB$ is isosceles. This means that P is the reflection of B across $\overline{AC}$, and thus $\triangle ACP \cong \triangle ACB$.

Now, we just need to find the area of $\triangle ACP$ and double it to find the area of $PABC$. $\triangle ACP$ has side lengths $AP = 16$ and $PC = AC = 17$. Dropping the altitude from C to $\overline{AP}$ to a point X splits $\triangle ACP$ into two $8 - 15 - 17$ right triangles. Thus, the area of $\triangle ACP = 8 \cdot 15 = 120$. So, the area of quadrilateral $PABC$ is $120 \cdot 2 = \boxed{240}$.

7. Square $ABCD$ has side length 4, and M is the point on segment $\overline{BC}$ such that $CM = 1$. A circle ω with radius r is constructed tangent to segments $\overline{AM}$, $\overline{AD}$, $\overline{CD}$. What is the area of ω?

Proposed by Thomas Yao.

Answer: $\boxed{\dfrac{16\pi}{9}}$

Solution: Extend lines AM and CD to meet at a point E. We now want to find the inradius of triangle $\triangle ADE$. Note that since $\overline{AB}$ and $\overline{CD}$ are parallel, we have $\angle MAB = \angle AED$. Furthermore, since $\angle ABM = \angle ADE = 90^\circ$, we know that $\triangle MBA$ is similar to $\triangle ADE$.

Now, triangle $\triangle MBA$ has side lengths $AB = 4$, $MB = 4 - 1 = 3$, and thus $AM = 5$ by the Pythagorean Theorem. The area of this triangle is $3 \cdot 4/2 = 6$, and the semiperimeter (half of the perimeter) is $s = (3 + 4 + 5)/2 = 6$, so the inradius of this triangle is given by $r = A/s = 6/6 = 1$. $\triangle MBA$ is similar to triangle $\triangle ADE$ by a scale factor of $4/3$ since $AD = 4$ and $MB = 3$, so the radius of ω is $1 \cdot 4/3 = 4/3$. Thus, our answer is $\pi \cdot (4/3)^2 = \boxed{\dfrac{16\pi}{9}}$.

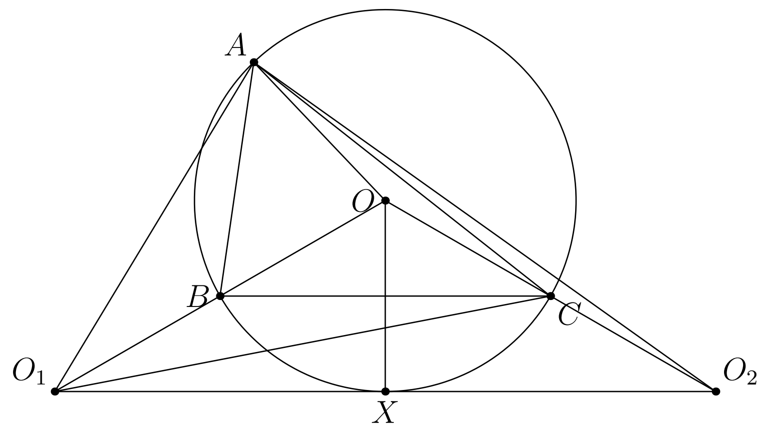
8. $\triangle ABC$ has side lengths $AB = 5$ and $AC = 8$. Let ω be the circle passing through A, B, and C, and let O be the center of ω. Let O_1 and O_2 be reflections of O across B and C, respectively. Given that segment $\overline{O_1O_2}$ is tangent to ω, what is the value of $[\triangle AO_1C] + [\triangle AO_2C]$?

($[\triangle ABC]$ *denotes the area of triangle* $\triangle ABC$)

Proposed by Bill Qian.

Answer: $\boxed{20\sqrt{3}}$

Solution:



Let the tangent point where O_1O_2 intersects ω be X, and let the radius of ω be R. Note that since O_2X is tangent to ω, we know that $\overline{OX} \perp \overline{O_1O_2}$. Consider triangle $\triangle OO_1X$. It is a right triangle with hypotenuse $OO_2 = 2 \cdot (OC) = 2R$, and a leg with length $OX = R$, so it must be a $30-60-90$ triangle. Similarly, $\triangle OO_2X$ is also a $30-60-90$ triangle. Thus, $\angle BOC = 120°$, so $\angle BAC = 60°$.

Now, note that we can split the area of $\triangle AO_1C$ as

$$[\triangle AO_1C] = [\triangle AOO_1] + [\triangle COO_1] + [\triangle AOC].$$

Thus,

$$\begin{aligned}[\triangle AO_1C] + [\triangle AO_2C] &= [\triangle AOO_1] + [\triangle COO_1] + [\triangle AOC] + [\triangle AO_1C] \\ &= [\triangle AOO_1] + [\triangle COO_1] + [\triangle AOO_2].\end{aligned}$$

Further, note that by reflection we know that $OO_1 = 2OB$ and $OO_2 = 2OC$, and that B is the midpoint of segment $\overline{OO_1}$ and that C is the midpoint of segment $\overline{OO_2}$. We can thus further split the triangles' areas into two congruent areas:

$$\begin{aligned}[\triangle AOO_1] &= [\triangle AOB] + [\triangle AO_1B] = 2 \cdot [\triangle AOB] \\ [\triangle AOO_2] &= [\triangle AOC] + [\triangle AO_2C] = 2 \cdot [\triangle AOC] \\ [\triangle COO_1] &= [\triangle COB] + [\triangle CO_1B] = 2 \cdot [\triangle BOC].\end{aligned}$$

Now, summing the areas,

$$\begin{aligned}[\triangle AO_1C] + [\triangle AO_2C] &= [\triangle AOO_1] + [\triangle COO_1] + [\triangle AOO_2] \\ &= 2 \cdot ([\triangle AOB] + [\triangle AOC] + [\triangle BOC]) \\ &= 2 \cdot [\triangle ABC].\end{aligned}$$

By the sine area formula, the area of $\triangle ABC$ is equal to $\frac{1}{2} \cdot 5 \cdot 8 \cdot \sin(60^\circ) = 20 \cdot \frac{\sqrt{3}}{2} = 10\sqrt{3}$, so the answer is $2 \cdot 10\sqrt{3} = \boxed{20\sqrt{3}}$.