1. Sam has 2 identical red coins, 2 identical blue coins, and 2 identical yellow coins. He picks 2 of the 6 coins to go shopping. If he can pick them in any order, how many combinations of colored coins can Sam pick?

Proposed by Kevin Shen.

Answer: $\boxed{6}$

Solution: Sam can either pick 2 coins of the same color, or 2 coins of different colors. There are 3 ways to pick 2 of the same color (RR, BB, YY), and 3 ways to pick 2 of different colors (RB, RY, BY), so the answer is $3 + 3 = \boxed{6}$.

2. Evan is playing a game that has easy levels and hard levels. He has a 50% chance to pass an easy level and a 10% chance to pass a hard level. What is the probability he passes 2 easy levels and 1 hard level, in that order, on his first try?

Proposed by Jeffery Boman.

Answer: $\boxed{\dfrac{1}{40}}$

Solution: Since each event is independent from each other, we multiply the probabilities of passing together to get $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{10} = \boxed{\dfrac{1}{40}}$.

3. The number 4167 is written on the board. Chris goes up to the board and erases two digits at random, leaving a two-digit number. What is the probability the remaining number is even?

Proposed by Kevin Shen.

Answer: $\boxed{\dfrac{1}{3}}$

Solution: Chris needs to erase 3 digits for the units digit of the number to be 4, so the units digit of his number to be 6. For this to happen, Chris needs to remove 7, and then either 4 or 1, so there are 2 ways to get an even number after removing two digits. There are $\binom{4}{2} = 4 \cdot 3/2 = 6$ ways to choose two digits to remove at random, so the answer is $\boxed{\dfrac{1}{3}}$.

4. 5 left-handed students and 3 right-handed students randomly choose seats from a row of 8 front-facing desks to take a test. What is the probability that no left-handed person sits directly to the right of a right-handed person?

Proposed by Evan Zhang.

Answer: $\boxed{\frac{1}{56}}$

Solution: The condition means that all right-handed people have only non-left handed people to their right. Since there are only left and right handed students, this means either the person to the right of a right-handed student is right-handed, or there is no one to their right. The students must then be sitting such that all right handed students are lined together to the right and all left handed students are thus on the left ($LLLLLRRR$). There are $5! \cdot 3!$ ways to arrange these distinct students. In total, there are $8!$ ways for the students to sit. Thus, the probability is

$$\frac{5! \cdot 3!}{8!} = \frac{1}{\binom{8}{3}} = \boxed{\frac{1}{56}}.$$

5. Thomas rolls a six-sided die 3 times and records the 3 numbers that he rolls. Given that the sum of the 3 numbers is 15, what is the probability he rolled at least one 6?

Proposed by Evan Zhang.

Answer: $\boxed{\frac{9}{10}}$

Solution: The only ways to roll a total of 15 is if either all three rolls are 5, if the rolls are 4, 5, and 6 in any of the $3! = 6$ ways, or if the rolls are 3, 6, and 6 in any of the $3!/2! = 3$ ways. This gives us $1 + 6 + 3 = 10$ total ways.

Using complementary counting, the only configuration which doesn't roll a 6 is when all dice rolled 5's. Consequently, the answer is

$$1 - \frac{1}{10} = \boxed{\frac{9}{10}}.$$

6. Three girls, Alice, Allison, and Avery, and three boys, Ben, Bill, and Yixuan, are sitting at a round table with six seats. If rotations are considered the same, how many distinct ways are there for the six people to sit around the table such that every boy sits next to at least one girl and every girl sits next to at least one boy?

Proposed by Kevin Shen.

Answer: $\boxed{84}$

Solution: Consider instead the complement. We wish to find the number of ways for the girls and boys to sit such that there is at least one boy that only sits next to boys, and at least one girl that only sits next to girls. This restricts the orientations to have all of the girls sitting next to each other, and all of the boys sitting next to each other. Within this orientation of GGG, BBB, there are $3! = 6$ ways to permute each girl and boy, so there are $6^2 = 36$ ways for them to sit.

To find the total number of orientations around the table, we can, without loss of generality, fix Alice to be at the top of the table. Then, there are $5! = 120$ ways to place the rest of the 5 people in the 5 remaining seats, and we don't have to worry about rotations since Alice is fixed at the top.

Thus, the answer is $120 - 36 = \boxed{84}$ ways.

7. Bo and Man are practicing archery, and can hit the target $2/3$ of the time and $1/3$ of the time, respectively. Each turn, they both shoot one arrow at the same time, playing until one player has hit the target two more times than the other. What is the expected number of turns until the game ends?

Proposed by Ashley Zhang.

Answer: $\boxed{\dfrac{90}{17}}$

Solution: Instead of keeping a running total of Bo and Man's scores, we can look at their point difference. For one player to gain 1 point difference, one player needs to hit the target and the other needs to miss. So, there is a

$$\frac{2}{3} \times \left(1 - \frac{1}{3}\right) + \frac{1}{3} \times \left(1 - \frac{2}{3}\right) = \frac{5}{9}$$

probability a point difference is earned on any given turn. Now, we can assume some player earns a point each turn and multiply our final expected value by $\frac{9}{5}$ to compensate.

Let E denote the number of points scored until one player wins. Note that given either Bo and Man win a point every turn, Bo has a

$$\frac{(2/3)\times(2/3)}{(5/9)} = \frac{4}{5}$$

and thus Man has a

$$1 - \frac{4}{5} = \frac{1}{5}$$

probability of gaining the point. After two turns, there are three possibilities: Bo wins with a $(4/5)^2 = 16/25$ chance, Man wins with a $(1/5)^2 = 1/25$ chance, and the game remains tied with a $2 \cdot (4/5) \cdot (1/5) = 8/25$ chance. Thus, we instantly win after 2 moves with probability 16/25, instantly win after 2 moves with probability 1/25, or we remain tied with probability 8/25 which increases the expected value by 2. We therefore have

$$\begin{aligned}
E &= \frac{16}{25}\cdot(2) + \frac{1}{25}\cdot(2) + \frac{8}{25}\cdot(E+2) \\
25E &= 34 + 8E + 16 \\
17E &= 50 \\
E &= \frac{50}{17}.
\end{aligned}$$

Finally, we multiply E by our previously determined factor of $\frac{9}{5}$ to get a final answer of

$$\frac{50}{17}E = \frac{50}{17}\cdot\frac{9}{5} = \boxed{\frac{90}{17}}.$$

8. A 5×5 square grid has each cell filled with either 1 or -1. A *move* consists of choosing one of the rows or columns and multiplying -1 to all the numbers in that row/column. From how many distinct starting grids is it possible to apply a finite number of *move*s so that all the cells in the board are filled with 1?

Proposed by Tane Park.

Answer: $\boxed{512}$

Solution: Consider an $n\times n$ board whose cells are filled entirely with 1. The problem is equivalent to finding the number of distinct grids that can be achieved after a transformation of this grid.

First, note that one *move* multiplies -1 the state of one cell. Since multiplication is commutative, we can thus apply a set of *move*s in any order and end up with the same resulting grid, as every cell will be multiplied by -1 the same number of times.

Because of this, and because applying 2 *moves* to one row/column results in no change of the board, we can assume without loss of generality that the number of *moves* done on a given row/column is either 0 or 1. Thus, with $2n$ total rows and columns, there are total of 2^{2n} possible combinations of *moves* possible. However, there may be one or more transformations which result in the same grid.

Since we are applying 0 or 1 *moves* to each column, we can say the value of a cell after transformation is $1 \cdot (-1)^c \cdot (-1)^r$, where c is the number of column moves and r is the number of row moves done on a cell. Note that this value is equal to $1 \cdot (-1)^{1-c} \cdot (-1)^{1-r}$, meaning that for a given transformation of each row/column r_i/c_i times, respectively, we may reach the same end result by transforming the rows/columns $(1 - r_i)/(1 - c_i)$ times, respectively.

Since r_i and c_i can only be 0 or 1, this must be the only way to achieve the same grid after a transformation $\{r, c\}$. Therefore, there are a total of $2^{2n}/2 = 2^{2n-1}$ distinct combinations of movements that result in a distinct board. Therefore, the answer is just 2^{2n-1}. Plugging in $n = 5$ gives $2^{2 \cdot 5 - 1} = 2^9 = \boxed{512}$.