1. What is the product of all x that satisfy $(x-1)^2 = 25$?

Proposed by Evan Zhang.

Answer: $\boxed{-24}$

Solution: Taking the square root of both sides, we get that $x - 1 = 5$ or $x - 1 = -5$. So, $x = 6$ or $x = -4$, so the answer is $6 \cdot (-4) = \boxed{-24}$.

2. Thomas buys 2 burgers and 3 fries from McWei's for \$29 dollars. Sam then buys 3 burgers and 2 fries from McWei's for \$31 dollars. How much would buying one burger and one fry cost, in dollars?

Proposed by Evan Zhang.

Answer: $\boxed{12}$

Solution: Let the cost of a burger be b and the cost of fry be f, we are looking for $b+f$. We are given

$$2b + 3f = 29 \quad \text{and} \quad 3b + 2f = 31.$$

Adding the equations together gives

$$\begin{aligned}(2b+3f) + (3b+2f) &= 29 + 31\\ 5b + 5f &= 60\\ b + f &= 12.\end{aligned}$$

Thus the cost of a burger and fry is $b + f = \boxed{12}$.

3. A positive number multiplied by its square root is 512. What is that number multiplied by its cube root?

Proposed by William Roe.

Answer: $\boxed{256}$

Solution: Let k be the desired number. Then

$$\begin{aligned}k \cdot \sqrt{k} = k \cdot k^{1/2} = k^{3/2} = 512\\ k = 512^{2/3} = \left(512^{\frac{1}{3}}\right)^2 = 8^2 = 64.\end{aligned}$$

Thus, our final answer is

$$k \cdot \sqrt[3]{k} = 64 \cdot (\sqrt[3]{64}) = 64 \cdot 4 = \boxed{256}.$$

4. Mr. Schwartz has a square-shaped bar of chocolate, broken into 45×45 congruent squares. Every day, he chooses one of the top, left, bottom, or right sides of the bar, and then eats the entire edge of squares corresponding to that side of the bar. If he has 455 congruent squares of chocolate remaining, how many days has it been since he started eating the chocolate bar?

Proposed by William Roe.

Answer: $\boxed{42}$

Solution: At every stage, Mr. Schwartz's chocolate bar is a rectangle. Each time he eats an edge of the chocolate bar, he subtracts 1 from either the width or height of the chocolate rectangle. So, we can write the dimensions of his rectangle after a certain period of time as $a \times b$, where a and b are less than or equal to 45. Note that the only factor pair of $455 = 5 \cdot 7 \cdot 13$ which satisfy this condition is $35 \cdot 13$. It must have taken $45 - 35 = 10$ days to achieve $a = 35$, and $45 - 13 = 32$ days to achieve $b = 13$, so the answer is $10 + 32 = \boxed{42}$ days.

5. How many ordered triplets of integers (x, y, z) satisfy $x + y = 12$ and $xz = y + z$?

Proposed by Sam Easaw.

Answer: $\boxed{4}$

Solution: Note that $x + y = 12$ implies $y = 12 - x$. Substituting into the second expression, $xz = (12 - x) + z$, and thus $xz - z + x = 12$. We can use Simon's Favorite Factoring trick:

$$xz - z + x - 1 = 12 - 1 = 11$$
$$z(x-1) + (x-1) = (z+1)(x-1) = 11.$$

Since x and z are integers, the $z + 1$ and the $x - 1$ in the expression may be any of the factor pairs of 11. There are 4 of these pairs: $(1, 11)$, $(11, 1)$, $(-1, -11)$, and $(-11, -1)$.

Since $xz = y + z$ and thus $y = xz - z$, we know that any (x, z) determines a unique y. Thus, the answer is $\boxed{4}$.

6. Let $P(x)$ be a quadratic polynomial with leading coefficient 1 such that the solutions to $P(x)^2 = 16$ are 1, 3, 5, and 7. Find $P(0)$.

Proposed by David Wang.

Answer: $\boxed{11}$

Solution: We can write $P(x) = x^2 + bx + c$. Note that the solutions of $P(x)^2 = 16$ are the solutions to $P(x) = 4$ combined with the solutions to $P(x) = -4$, or thus the roots of polynomials $P(x) - 4$ and $P(x) + 4$, which are both $P(x)$ translated vertically by 4 units.

Since $P(x) - 4$ and $P(x) + 4$ are just parabolas translated vertically by 4, they thus have the same x-value of their vertex.

Note that the vertex of $P(x)$ is the average of its two roots. The only way the two polynomials' roots can average to the same value is for one quadratic to have roots of 3 and 5, and the other to have roots of 1 and 7. Thus, the polynomials are

$$(x-3)(x-5) = x^2 - 8x + 15 \quad \text{and} \quad (x-1)(x-7) = x^2 - 8x + 7.$$

So, we must have

$$P(x) = x^2 - 8x + 15 - 4 = x^2 - 8x + 7 + 4 = x^2 - 8x + 11,$$

and $P(0) = \boxed{11}$.

7. Katherine is studying a polynomial $f(x) = x^4 - ax^2 + b$. She wants to choose an ordered pair of real numbers (a, b) so that $a + b = 169$ and all roots of $f(x)$ are integers. What is the sum of all possible a she can choose?

Proposed by Sam Easaw.

Answer: $\boxed{194}$

Solution: First, note that $f(x)$ is even, as it is the sum of even powers of x, so $f(x) = f(-x)$. This means that for any root at $x = r$ of f, there exists another root at $x = -r$. So, the roots of f must be r, s, $-r$, and $-s$, in some order for some integers r and s, and thus f can be written as

$$f(x) = (x-r)(x-s)(x+r)(x+s).$$

Consider the function f evaluated at $i = \sqrt{-1}$:

$$f(i) = (i)^4 - a(i)^2 + b = 1 - a(-1) + b = 1 + a + b = 1 + (169) = 170.$$

Now, we also know that

$$\begin{aligned} f(i) &= (i-r)(i-s)(i+r)(i+s) \\ &= (i-r)(i+r)(i-s)(i+s) \\ &= ((i)^2 - r^2)((i)^2 - s^2) = (-1-r^2)(-1-s^2), \end{aligned}$$

so finally $f(i) = (1 + r^2)(1 + s^2) = 170$.

Now, consider that $f(x)$ can be written as $f(x) = (x^2)^2 - a(x^2) + b = y^2 - ay + b$ for $y = x^2$. By Vieta's formulas, a will be the sum of the roots of this polynomial. The roots of this polynomial must be $(r)^2 = (-r)^2 = r^2$ and $(s)^2 = (-s)^2 = s^2$, so $a = r^2 + s^2$.

Since r and s are integers, $1 + r^2$ and $1 + s^2$ must be integers as well. We can now check based on the factors of $f(i) = 170$:

- If $1 + r^2 = 1$ and $1 + s^2 = 170$, then $r = 0$ and $s = 13$. This means that $a = (0)^2 + (13)^2 = 169$.
- If $1 + r^2 = 10$ and $1 + s^2 = 17$, then $r = 3$ and $s = 4$, so $a = 3^2 + 4^2 = 25$.

It can be verified that there are no other factor pairs which generate integer r and s, so the answer is $169 + 25 = \boxed{194}$.

Alternate Solution: Consider that, by Vieta's formulas as above, we find that $a = r^2 + s^2$ and $b = r^2 \cdot s^2$. Plugging into the relation $a + b = 169$, we get

$$\begin{aligned} a + b = r^2 + s^2 + r^2s^2 &= 169 \\ r^2s^2 + r^2 + s^2 + 1 &= 169 + 1 \\ (r^2 + 1)(s^2 + 1) &= 170 \end{aligned}$$

using Simon's Favorite Factoring Trick. We can proceed with the above solution from here.

8. Let a, b and c be positive real numbers that satisfy

$$\begin{aligned} 2026ab &= c^2 + 1 \\ 2026ac &= 8b^2 + 2 \\ 2026bc &= \frac{a^2}{8} + \frac{1}{2}. \end{aligned}$$

What is the value of $a + b + c$?

Proposed by David Wang.

Answer: $\boxed{\dfrac{7}{90}}$

Solution: Multiplying the equations by c, b and a respectively, our equations become

$$\begin{aligned}2026abc &= c^3 + c\\ 2026abc &= 8b^3 + 2b\\ 2026abc &= \frac{a^3}{8} + \frac{a}{2}.\end{aligned}$$

Combining equations 1 and 2, we get

$$\begin{aligned}2026abc = c^3 + c = 8b^3 + 2b &\implies 8b^3 - c^3 + 2b - c = 0\\ &\implies (2b-c)(4b^2 + 2bc + c^2 + 1) = 0.\end{aligned}$$

As a, b and c are all positive, $4b^2 + 2bc + c^2 + 1 > 0$, so we know that $2b - c = 0$, or $c = 2b$.

Similarly, by combining equations 1 and 3, we get

$$\begin{aligned}2026abc = c^3 + c = \frac{a^3}{8} + \frac{a}{2} &\implies c^3 - \frac{a^3}{8} + c - \frac{a}{2} = 0\\ &\implies \left(c - \frac{a}{2}\right)\left(c^2 + \frac{ca}{2} + \frac{a^2}{4} + 1\right) = 0\\ &\implies c - \frac{a}{2} = 0 \implies c = \frac{a}{2} \implies 2c = a.\end{aligned}$$

Thus, we have

$$2026(2c)\left(\frac{c}{2}\right) = c^2 + 1 \implies 2025c^2 = 1 \implies c = \frac{1}{45}.$$

Solving for a and b gives

$$\begin{aligned}2c = a &\implies a = \frac{2}{45}\\ c = 2b &\implies b = \frac{1}{90}\\ &\implies a + b + c = \frac{2}{45} + \frac{1}{90} + \frac{1}{45} = \boxed{\frac{7}{90}}.\end{aligned}$$