1. Alice has a square cookie of side length 8 inches, and Bill has a rectangular cookie of length 4 inches and unknown width. If the area of Alice and Bill's cookies are the same, what is the width of Bill's cookie?

Proposed by Sam Easaw.

Answer: $\boxed{16}$

Solution: Alice's square has area $8^2 = 64$. Bill's rectangle has area $4a = 64$. So, $a = \frac{64}{4} = \boxed{16}$.

2. Evan is running along the perimeter of a circle of radius 1. How far does he have to run to complete 12 laps around the circle?

Proposed by Sam Easaw.

Answer: $\boxed{24\pi}$

Solution: The circumference of a circle is given by $2\pi \cdot r$, where r is the radius. Each lap has a length of $2\pi \cdot 1 = 2\pi$, so 12 laps will have Evan run a length of $12 \cdot 2\pi = \boxed{24\pi}$.

3. Four points A, B, C, and D, lie on a line in some order and satisfy $BC = 4$, $AC = 6$, and $BD = 12$. What is the largest possible value of AD?

Proposed by Sam Easaw.

Answer: $\boxed{22}$

Solution: Start by drawing BD on a line. Then, C must lie within 4 units of B in either direction, and A must lie within 6 units of C in either direction, so we place C and further A as far away from D as possible. This results in a line of points A, C, B, D, or reversed. Thus, $AD = AC + BC + BD = 12 + 6 + 4 = \boxed{22}$.

4. Square $ABCD$ has side length 3, and points E and F are chosen on segment $\overline{BC}$ and $\overline{CD}$ such that $CE = CF = 1$. What is the area of triangle $\triangle AEF$?

Proposed by Sam Easaw.

Answer: $\boxed{\dfrac{5}{2}}$

Solution: Instead of finding the area of $\triangle AEF$ directly, we can subtract out the areas of the other triangles in the square. The area of $\triangle ADF$ is $2 \cdot 3/2 = 3$, which is the same as the area of triangle $\triangle ACE$. The area of triangle $\triangle CEF$ is $1 \cdot 1/2 = \frac{1}{2}$. The area of the entire square is $3 \cdot 3 = 9$. Thus, the area of $\triangle AEF$ is

$$9 - 3 - 3 - \frac{1}{2} = \boxed{\frac{5}{2}}.$$

5. Sam, David, and Bill are standing at points S, D, and B around a circle with diameter $\overline{BD}$. They are standing in such a way that $BD = 13$ and $SD = 5$. What is the shortest distance Bill has to run if he wants to run directly to Sam and then directly to David?

Proposed by Christopher Wang.

Answer: $\boxed{17}$

Solution: Let David be at a point D, Bill be at a point B, and Sam be at a point S. Since BD is a diameter of the circle, we know that $\angle BSD = 90°$, since it subtends half of the circle. So, since $BD = 13$ and $SD = 5$, we know that $BS = \sqrt{13^2 - 5^2} = 12$. Thus, the answer is $BS + SD = 12 + 5 = \boxed{17}$.

6. Let P be a hexagon of side length 2. Andy draws a flower-shape onto the hexagon by constructing a circle of radius 1 at each vertex of P. What is the perimeter of Andy's new closed flower?

Proposed by Siyuan Yan.

Answer: $\boxed{8\pi}$

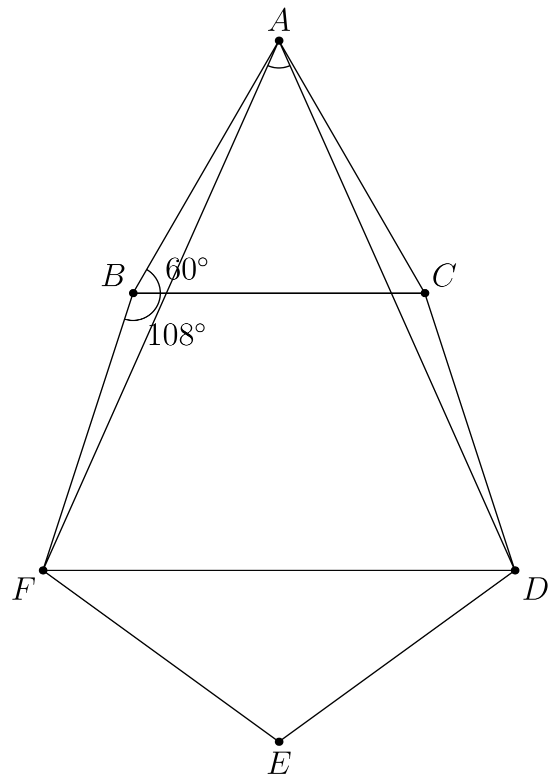
Solution: Since the hexagon has side length 2, and the circles have radius 1, all of the circles must be tangent to each other, so we just need to find the perimeter of the circles that lie outside the hexagon. Since each angle in a hexagon is $120°$, each circle has $120°$ of circumference lying inside of the circle, and thus $360° - 120° = 240°$ of circumference lying outside of it. This is $\frac{240°}{360°} = \frac{2}{3}$ of circumference for each circle. Since there are 6 circles, we have $\frac{2}{3} \cdot 6 = 4$ circumferences of one circle in total. Thus, the answer is $4 \cdot (2 \cdot \pi \cdot 1) = \boxed{8\pi}$.

7. Equilateral triangle $\triangle ABC$ has a side length of 1. A regular pentagon $BCDEF$ is constructed on the exterior of $\triangle ABC$. In degrees, what is the measure of angle $\angle DAF$?

Proposed by Sam Easaw.

Answer: $\boxed{48^\circ}$

Solution:



Refer to above diagram. We know that each interior angle in an equilateral triangle measures 60°, and in a regular pentagon measures $(5-2)\cdot 180^\circ/5 = 108^\circ$. Note that $\triangle ABF$ is isosceles, and $\angle ABF = 60^\circ + 108^\circ = 168^\circ$, so the base angles each measure $\frac{1}{2}(180^\circ - 168^\circ) = 12^\circ/2 = 6^\circ$. Now, the measure of $\angle DAF$ is equal to $60^\circ - \angle BAF - \angle CAD = 60^\circ - 2\cdot\angle BAF$ by symmetry, so $\angle DAF = 60^\circ - 2\cdot 6^\circ = \boxed{48^\circ}$.

8. Triangle $\triangle ABC$ has $AB = 16$ and $AC = 17$. There exists a point P outside $\triangle ABC$ such that $AP = 16$, $CP = 17$, and $\overline{AC}$ is perpendicular to $\overline{PB}$. What is the area of quadrilateral $PABC$?

Proposed by Sam Easaw.

Answer: $\boxed{240}$

Solution: Note that $PC = AC = 17$, so triangle $\triangle PCA$ is isosceles. Also, $AB = AP = 16$, so $\triangle PBA$ is also isosceles. Now, since $\overline{AC}$ is perpendicular to $\overline{PB}$, we know that $\overline{AC}$ is the altitude of $\triangle PAB$, which is also the angle bisector of $\angle BAP$ since $\triangle PAB$ is isosceles. This means that P is the reflection of B across $\overline{AC}$, and thus $\triangle ACP \cong \triangle ACB$.

Now, we just need to find the area of $\triangle ACP$ and double it to find the area of $PABC$. $\triangle ACP$ has side lengths $AP = 16$ and $PC = AC = 17$. Dropping the altitude from C to $\overline{AP}$ to a point X splits $\triangle ACP$ into two $8 - 15 - 17$ right triangles. Thus, the area of $\triangle ACP = 8 \cdot 15 = 120$. So, the area of quadrilateral $PABC$ is $120 \cdot 2 = \boxed{240}$.