

1. Soy milk is sold in packs of 8 cans. If Bill wants 60 cans of soy milk, what is the least number of packs he has to buy?

Proposed by Sam Easaw.

Answer: 8

Solution: Bill can only buy multiples of 8 cans. The smallest multiple of 8 greater than 60 is 64, so $\frac{64}{8} = \boxed{8}$.

2. What is the product of all x that satisfy $(x - 1)^2 = 25$?

Proposed by Evan Zhang.

Answer: -24

Solution: Taking the square root of both sides, we get that $x - 1 = 5$ or $x - 1 = -5$. So, $x = 6$ or $x = -4$, so the answer is $6 \cdot (-4) = \boxed{-24}$.

3. Bill has a collection of 24 math, physics, and chemistry books. If $\frac{1}{2}$ of Bill's books are about math, and $\frac{1}{3}$ of Bill's books are about physics, how many of Bill's books are about chemistry?

Proposed by Sam Easaw.

Answer: 4

Solution: The fraction of Bill's books that are about chemistry is $1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$. So, Bill has $24 \cdot \frac{1}{6} = \boxed{4}$ chemistry books.

4. Thomas buys 2 burgers and 3 fries from McWei's for \$29 dollars. Sam then buys 3 burgers and 2 fries from McWei's for \$31 dollars. How much would buying one burger and one fry cost, in dollars?

Proposed by Evan Zhang.

Answer: 12

Solution: Let the cost of a burger be b and the cost of fry be f , we are looking for $b + f$. We are given

$$2b + 3f = 29 \quad \text{and} \quad 3b + 2f = 31.$$

Adding the equations together gives

$$(2b + 3f) + (3b + 2f) = 29 + 31$$

$$5b + 5f = 60$$

$$b + f = 12.$$

Thus the cost of a burger and fry is $b + f = \boxed{12}$.

5. Peterfun is going on a journey around the world. If he spends 30 days boating at 150 miles/day and 50 days boating at 102 miles/day, what is his average speed over his entire journey?

Proposed by Jeffrey Boman.

Answer: $\boxed{120}$

Solution: His average speed over the journey is the total distance he traveled over the total time he traveled for. Note that distance = rate $\cdot$ time, so the distance is equal to the rate of travel multiplied by the time it took to travel. The first distance he spent boating was $150 \cdot 30 = 4500$ miles, and the second distance he spent boating was $102 \cdot 50 = 5100$ miles. In total, he spent 80 days boating, so the average speed is

$$\frac{(102 \cdot 50 + 30 \cdot 150)}{80} = \frac{4500 + 5100}{80} = \frac{9600}{80} = \boxed{120} \text{ miles/day.}$$

6. Mr. Schwartz has a square-shaped bar of chocolate, broken into 45×45 congruent squares. Every day, he chooses one of the top, left, bottom, or right sides of the bar, and then eats the entire edge of squares corresponding to that side of the bar. If he has 455 congruent squares of chocolate remaining, how many days has it been since he started eating the chocolate bar?

Proposed by William Roe.

Answer: $\boxed{42}$

Solution: At every stage, Mr. Schwartz's chocolate bar is a rectangle. Each time he eats an edge of the chocolate bar, he subtracts 1 from either the width or height of the chocolate rectangle. So, we can write the dimensions of his rectangle after a certain period of time as $a \times b$, where a and b are less than or equal to 45. Note that the only factor pair of $455 = 5 \cdot 7 \cdot 13$ which satisfy this condition is $35 \cdot 13$. It must have taken $45 - 35 = 10$ days to achieve $a = 35$, and $45 - 13 = 32$ days to achieve $b = 13$, so the answer is $10 + 32 = \boxed{42}$ days.

7. How many ordered triplets of integers (x, y, z) satisfy $x + y = 12$ and $xz = y + z$?

Proposed by Sam Easaw.

Answer: $\boxed{4}$

Solution: Note that $x + y = 12$ implies $y = 12 - x$. Substituting into the second expression, $xz = (12 - x) + z$, and thus $xz - z + x = 12$. We can use Simon's Favorite Factoring trick:

$$\begin{aligned}xz - z + x - 1 &= 12 - 1 = 11 \\z(x - 1) + (x - 1) &= (z + 1)(x - 1) = 11.\end{aligned}$$

Since x and z are integers, the $z + 1$ and the $x - 1$ in the expression may be any of the factor pairs of 11. There are 4 of these pairs: $(1, 11)$, $(11, 1)$, $(-1, -11)$, and $(-11, -1)$.

Since $xz = y + z$ and thus $y = xz - z$, we know that any (x, z) determines a unique y . Thus, the answer is $\boxed{4}$.

8. Let $P(x)$ be a quadratic polynomial with leading coefficient 1 such that the solutions to $P(x)^2 = 16$ are 1, 3, 5, and 7. Find $P(0)$.

Proposed by David Wang.

Answer: $\boxed{11}$

Solution: We can write $P(x) = x^2 + bx + c$. Note that the solutions of $P(x)^2 = 16$ are the solutions to $P(x) = 4$ combined with the solutions to $P(x) = -4$, or thus the roots of polynomials $P(x) - 4$ and $P(x) + 4$, which are both $P(x)$ translated vertically by 4 units.

Since $P(x) - 4$ and $P(x) + 4$ are just parabolas translated vertically by 4, they thus have the same x -value of their vertex.

Note that the vertex of $P(x)$ is the average of its two roots. The only way the two polynomials' roots can average to the same value is for one quadratic to have roots of 3 and 5, and the other to have roots of 1 and 7. Thus, the polynomials are

$$(x - 3)(x - 5) = x^2 - 8x + 15 \quad \text{and} \quad (x - 1)(x - 7) = x^2 - 8x + 7.$$

So, we must have

$$P(x) = x^2 - 8x + 15 - 4 = x^2 - 8x + 7 + 4 = x^2 - 8x + 11,$$

and $P(0) = \boxed{11}$.