MBMT Number Theory Round — Brahmagupta

March 9, 2024

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

1 A car is falling at a constant rate of 5 meters per second. A bus is falling at a constant rate of 7.5 meters per second. If they start at the same height, what is the difference in their heights in 1 minute? There are 60 seconds in a minute.

2 The seniors at Montgomery Blair High School are going on a field trip. There will be 200 students and 25 teachers on the trip. Each bus can carry 45 passengers. How many buses will be needed?

3 The sum of two primes is 31. What is the value of the larger prime? A prime number is a number whose only divisors are one and itself (one is not a prime).

4 A number is called relatively prime to another number if they share no factors other than 1. How many positive integers less than 23 are relatively prime to 23?

5 Find the number of divisors of 2024.

6 If x, y are nonnegative integers, and $xy + x + 3y = 1$, find $x + y$.

7 Manny likes mung beans. He can purchase 20 beans for \$3, 50 beans for \$7, and 160 beans for \$19. Manny needs at least 770 beans. What is the minimum amount of money he needs to spend?

8 A positive integer is "inspirational" if it has at least three factors and the sum of its three smallest positive factors is 12. How many inspirational numbers are less than 2024?