MBMT Algebra Round — Germain

April 16, 2023

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** Yunyi discovers that when he multiplies a number by 2, he gets the same result as if he added 5 to the number. What is the number?

_____ **2** At noon, a clock shows the correct time. At m minutes after noon, the clock suddenly starts ticking at half the speed it should. At 1:30, the clock shows 1 o'clock. Find the value of m.

_____ **3** Compute $\sqrt{10004 \cdot 9996 + 16}$.

_____ **4** Let a_n be the sum of integers from 1 to n (for instance, $a_1 = 1$, $a_3 = 1 + 2 + 3$). And let $b_n = \frac{a_{2n-1}}{a_{2n}}$. Find $b_1 \cdot b_2 \cdot b_3 \cdot \ldots \cdot b_{10}$.

_____ **5** Suppose Bradley has a sequence such that $x_n = 3x_{n-1} + 2$. If $x_0 = 0$, then what is x_{20}?

_____ **6** Compute $0.25^{0.25^{0.25^{\cdots}}}$ where the number of 0.25's goes to infinity.

_____ **7** A function of the form $f(x) = ax^3 + bx^2 + cx + d$ is known to have $f(100) = 2, f(101) = 0, f(102) = 2, f(103) = 3$. Find $f(104)$.

_____ **8** Suppose x, y, z are positive reals that satisfy $2x + 3y + 4z = 12$ and $3xyz = 8$. What is $x + y + z$?