MBMT Number Theory Round — Zermelo

May 21, 2022

Full Name _____

Student ID Number _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round consists of **8** questions. You will have **30** minutes to complete the round. Each question is *not* worth the same number of points. Questions answered correctly by fewer competitors will be weighted more heavily. Please write your answers in a reasonably simplified form.

_____ **1** What is the largest integer less than 100 that is not divisible by 2, 3, or 5?

_____ **2** Find the largest three digit integer which has an odd sum of digits, and an even product of digits.

_____ **3** How many zeros does $5! + 10! + 15! + 20! + 25!$ end in? Recall that $n! = 1 \cdot 2 \cdot \cdots \cdot n$.

_____ **4** Suppose a, b, and c are equal to $2, 3$ and 4, in some order. What's the last digit of the greatest possible value of a^{b^c}?

_____ **5** Let S be the set of all even integers greater than or equal to 2022. What's the unique element n of S such that the number of divisors of $512n$ that aren't divisors of 512 is minimized?

_____ **6** In a regular 10 by 10 multiplication table, the numbers that would appear are the products ab for every a ranging from 1 to 10 and every b ranging from 1 to 10.

A wrong multiplication table is a multiplication table that only keeps the last digit of the product instead of entire product. In a 10 by 10 wrong multiplication table starting from 1, what is the least number of times that any result appears?

_____ **7** Find the number of ordered pairs of positive integers (a, b) such that the least common multiple of a and b is $13^{29} \cdot 29^{13}$.

_____ **8** Two items have prices $\$a.bc$ and $\$d.ef$ for digits a, b, c, d, e, f. When the cashier finds their value, he gets the same result regardless if he added them or multiplied them. Find the largest possible value of the digit d.