MBMT Zermelo Guts Round — Set 1

May 21, 2022

1 **[3]** What is $1 + 2 \cdot 3$?

2 **[3]** A square of side length 2 is cut into 4 congruent squares. What is the perimeter of one of the 4 squares?

3 **[3]** 6 people split a bag of cookies such that they each get 21 cookies. Kyle comes and demands his share of cookies. If the 7 people then re-split the cookies equally, how many cookies does Kyle get?

4 **[3]** Blobby flips 4 coins. What is the probability he sees at least one heads and one tails?

5 **[3]** The product of 10 consecutive positive integers ends in 3 zeros. What is the minimum possible value of the smallest of the 10 integers?

MBMT Zermelo Guts Round — Set 2

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6 **[4]** How many perfect squares less than 100 are odd?

7 **[4]** Guuce continually rolls a fair 6-sided dice until he rolls a 1 or a 6. He wins if he rolls a 6, and loses if he rolls a 1. What is the probability that Guuce wins?

8 **[4]** The perimeter and area of a square with integer side lengths are both three digit integers. How many possible values are there for the side length of the square?

9 **[4]** In the coordinate plane, a point is selected in the rectangle defined by $-6 \le x \le 4$ and $-2 \le y \le 8$. What is the largest possible distance between the point and the origin, $(0, 0)$?

10 **[4]** The sum of two numbers is 6 and the sum of their squares is 32. Find the product of the two numbers.

MBMT Zermelo Guts Round — Set 3

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11 **[5]** How many two digit numbers are there such that the product of their digits is prime?

12 **[5]** Triangle ABC has area 4 and $\overline{AB} = 4$. What is the maximum possible value of $\angle ACB$?

13 **[5]** Let $ABCD$ be an iscoceles trapezoid with $AB = CD$ and M be the midpoint of $\overline{AD}$. If $\triangle ABM$ and $\triangle MCD$ are equilateral, and $BC = 4$, find the area of trapezoid $ABCD$.

14 **[5]** Let x and y be positive real numbers that satisfy $(x^2 + y^2)^2 = y^2$. Find the maximum possible value of x.

15 **[5]** Let AOB be a quarter circle with center O and radius 4. Let ω_1 and ω_2 be semicircles inside AOB with diameters OA and OB, respectively. Find the area of the region within AOB but outside of ω_1 and ω_2.

MBMT Zermelo Guts Round — Set 4

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16 **[7]** Integers a, b, c form a geometric sequence with an integer common ratio. If $c = a + 56$, find b.

17 **[7]** In parallelogram $ABCD$, $\angle A \cdot \angle C - \angle B \cdot \angle D = 720^\circ$ where all angles are in degrees. Find the value of $\angle C$.

18 **[7]** Steven likes arranging his rocks. A mountain formation is where the sequence of rocks to the left of the tallest rock increase in height while the sequence of rocks to the right of the tallest rock decrease in height. If his rocks are $1, 2, \ldots, 10$ inches in height, how many mountain formations are possible? For example: the sequences (1-3-5-6-10-9-8-7-4-2) and (1-2-3-4-5-6-7-8-9-10) are considered mountain formations.

19 **[7]** Find the smallest 5-digit multiple of 11 whose sum of digits is 15.

20 **[7]** Two circles, ω_1 and ω_2, have radii of 2 and 8, respectively, and are externally tangent at point P. Line l is tangent to the two circles, intersecting ω_1 at A and ω_2 at B. Line m passes through P and is tangent to both circles. If line m intersects line l at point Q, calculate the length of PQ.

MBMT Zermelo Guts Round — Set 5

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21 **[9]** Sen picks a random 1 million digit integer. Each digit of the integer is placed into a list. The probability that the last digit of the integer is strictly greater than twice the median of the digit list is closest to $\frac{1}{a}$, for some integer a. What is a?

22 **[9]** Let 6 points be evenly spaced on a circle with center O, and let S be a set of 7 points: the 6 points on the circle and O. How many equilateral polygons (not self-intersecting and not necessarily convex) can be formed using some subset of S as vertices?

23 **[9]** For a positive integer n, define r_n recursively as follows: $r_n = r_{n-1}^2 + r_{n-2}^2 + \ldots + r_0^2$, where $r_0 = 1$. Find the greatest integer less than

$$\frac{r_2}{r_1^2} + \frac{r_3}{r_2^2} + \ldots + \frac{r_{2023}}{r_{2022}^2}.$$

24 **[9]** Arnav starts at 21 on the number line. Every minute, if he was at n, he randomly teleports to $2n^2, n^2$, or $\frac{n^2}{4}$ with equal chance. What is the probability that Arnav only ever steps on integers?

25 **[9]** Let $ABCD$ be a rectangle inscribed in circle ω with $AB = 10$. If P is the intersection of the tangents to ω at C and D, what is the minimum distance from P to AB?

MBMT Zermelo Guts Round — Set 6

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26 **[12]** For every person who wrote a problem that appeared on the final MBMT tests, take the number of problems they wrote, and then take that number's factorial, and finally multiply all these together to get n. Estimate the greatest integer a such that 2^a evenly divides n.

27 **[12]** Circles of radius 5 are centered at each corner of a square with side length 6. If a random point P is chosen randomly inside the square, what is the probability that P lies within all four circles?

28 **[12]** Mr. Rose's evil cousin, Mr. Caulem, has teaches a class of three hundred bees. Every week, he tries to disrupt Mr. Rose's 4th period by sending three of his bee students to fly around and make human students panic. Unfortunately, no pair of bees can fly together twice, as then Mr. Rose will become suspicious and trace them back to Mr. Caulem. What's the largest number of weeks Mr. Caulem can disrupt Mr. Rose's class?

29 **[12]** Two blind brothers Beard and Bored are driving their tractors in the middle of a field facing north, and both are 10 meters west from a roast turkey. Beard, can turn exactly $0.7°$ and Bored can turn exactly $0.2°$ degrees. Driving at a consistent 2 meters per second, they drive straight until they notice the smell of the turkey getting farther away, and then turn right and repeat until they get to the turkey.

Suppose Beard gets to the Turkey in about 818.5 seconds. Estimate the amount of time it will take Bored.

30 **[12]** Let a be the probability that 4 randomly chosen positive integers have no common divisor except for 1. Estimate $300a$. Note that the integers $1, 2, 3, 4$ have no common divisor except for 1.

Remark. This problem is asking you to find

$$300 \lim_{n\to\infty} a_n,$$

if a_n is defined to be the probability that 4 randomly chosen integers from $\{1, 2, ..., n\}$ have greatest common divisor 1.